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Luigi Brighi and Marcello D'Amato*

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Abstract

We study a two periods entry game where the incumbent firm, who has private information about his own production costs, makes a non observable long run investment choice, along with a pricing decision observed by the entrant. The investment choice affects both post-entry competition and first period cost of production, so that the cost of signaling becomes endogenous. The game is solved following Bayes-Nash requirements, the intuitive criterion is used to constrain off-equilibrium beliefs. When investment is publicly observable, it is shown that the unique intuitive equilibrium is the separating equilibrium with limit pricing and no entry deterrence. When investment is not observable, quite remarkably, there exists a unique intuitive pooling equilibrium which is Pareto superior, from the incumbent's point of view, to the unique intuitive separating equilibrium. In the pooling equilibrium no entry takes place and the price is below the low cost monopoly price. Thus, when investment is secret, a limit pricing policy supports entry deterrence. Our model provides an example of secret barriers to entry and their relationship with limit pricing. We also contribute to the analysis of a relatively under-researched class of games where the cost of signaling unobservable characteristics is endogenously determined by unobserved actions.

JEL Classification Numbers: D58, L51.
Keywords: Entry deterrence, limit pricing, signaling, pooling equilibrium.

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1 Introduction

In the context of a classical entry game with private information of the incumbent about his costs (Milgrom and Roberts, 1980, 1982) we introduce the possibility of investment in an otherwise standard signaling model. The main motivation for our investigation is provided by a specific interest in the theory of limit pricing and its relationship with barriers to entry. Indeed one of the original formulations highlighted already the presumption that

> *the potential entrant regards the current pricing policy of established sellers as being probably a statement of intentions rather than a bluff*, (Bain, 1949, p. 453).

The credible statement of future intentions built in the current pricing policy may pertain to a persistent state variable (e.g. incumbent's costs as in Milgrom and Roberts, 1982) but it can also pertain to persistent effects of incumbent's investment choices. The analysis of the effects of publicly irreversible decisions affecting long run competition and entry conditions is well developed and quite influential for business strategy analysis[1] and antitrust purposes. However, a more secretive character than that required to ensure the commitment value of a public irreversible long run decision can be a feature of many choices made by the firm about the production process, such as the exploitation of the learning curve, investments in R&D, or about long run contracts. Studying if and how the incumbent's pricing strategy will convey information about the performed investment or a long run secret contract, and hence about the profitability of entry, is the specific aim of the present investigation.

To address these issues we consider a straightforward extension of the standard entry game with private information on production costs where an incumbent may also have the opportunity to secretly invest in a cost reducing technology. After secret investment has been undertaken and pricing decisions have been made and publicized to the market, the entry decision by the potential entrant is taken. If entry takes place the post-entry stage is the standard Cournot-Nash outcome, with learning upon

[1] See Fudenberg and Tirole (1984) and Tirole, (1988).

entry so that current pricing only affects the entry decision and not the post entry game due to learning [2].

Our analysis unveils an important consequence of investment unobservablity for limit pricing strategies. In a precise sense, equilibrium limit pricing can establish a commitment value of secret investment and, more precisely, that equilibrium limit pricing limits entry by signaling a sufficiently large secret investment. Interestingly this outcome could not have been achieved if the same game had been played under complete information of the entrant about the incumbent's type. More importantly, we show that entry deterrence obtained under private information and unobservable investment would not obtain in the predicted equilibrium outcome of the game in the presence of private information when investment is observable.

Two elements considered in the model can be highlighted here to understand these results. Compared to the standard entry game with private information, the entrant strives to infer both the incumbent's type, an exogenous characteristics, and the incumbent action (investment undertaken, an endogenous variable) since both pieces of information are relevant and taken into account at the moment when entry is decided. Which one of the two bits of information a pricing policy is able to convey, and to what extent, is the new issue that will be carefully examined. The second feature of our model is that signaling costs are endogenous to the incumbent: by reducing the cost in both periods, investment will not only affect post-entry competition, but will also lower the cost of signaling associated with pre-entry pricing policy. Thus, the cost of signaling becomes endogenous and the incentives to invest are intertwined with the incentives to signal faced by both types of incumbents.

Our analysis is performed under a few assumptions on the structural elements of the industry in the aim to be as close as possible to the classical model of entry, so that the key implications of the presence of secret investments for limit pricing are better uncovered. Specifically, we assume that there are only two types of incumbents, the high cost type and the low cost type; the low cost type is assumed to be on the technology frontier so that he gains no direct benefit by further investing in cost reducing activities. The high cost type, instead, can decide to modulate the

[2] As in Milgrom and Roberts, (1982).

level of effort in cost reducing activities taking into account its impact on both the entry decision and the post-entry market. Entry is always profitable by the potential entrant facing a high cost type of incumbent, no matter what level of investment has been undertaken. In contrast, entry is unprofitable in the face of a low cost incumbent. Hence, in the context of the above model, the key question is whether entry can be credibly deterred, whenever there exists a threshold level of investment such that the expected profits to the entrant are zero when the incumbent's costs are private information. A related question is whether the observability of investment makes any difference for the plausible outcome of the game. Notice that the above assumptions guarantee that the possibility to invest for the high cost type would not enable the incumbent to raise entry barriers in the same game played under complete information so that the market will be contested whenever the established firm is of a high cost type.

In our analysis we show that when the incumbent's type is private information, but investment is directly observable by the potential entrant, a unique intuitive separating equilibrium emerges, where the low cost incumbent prices below his monopoly level and where entry takes place when the incumbent has high costs. Pooling equilibria are possible, but they do not survive the intuitive criterion and they are not plausible outcomes of the entry game. Thus, when investment is publicly observable, limit pricing does not deter entry.

When the incumbent's type is private information and, instead, investment is secret, both a unique separating and a unique pooling equilibrium pass the intuitive criterion. Most importantly, the pooling equilibrium is Pareto superior from the point of view of the established firms and hence the pooling, rather than the separating equilibrium, seems to be the most plausible solution of the entry game. In the pooling equilibrium entry never takes place and both types of incumbent play a limit price below the monopoly price of the low cost incumbent. Quite surprisingly, unobservable investment takes on the power to deter entry through the pricing policy, a property that investment does not have when it is itself publicly observable. In a precise sense, limit pricing confers investment a commitment value: whenever there exists an entry deterrence level of secret investment the pooling equilibrium pricing strategies will support it.

The above results are derived by introducing some adaptations of existing techniques and solution concepts for standard signaling models to the case in which the sender's strategy set also includes private choices. The notion of Perfect Bayesian Equilibrium is applied by taking into account that the entrant not only formulates expectations about the sender's type (costs), but also about the sender's action (investment) and by requiring that expectations are consistent with the incumbent's choices, i.e. Bayes rule applies whenever possible. The intuitive criterion (Cho and Kreps, 1987) is used to deal with the multiplicity of equilibria.

Our model is very much related to the classical analysis of the battle for market shares (Roberts, 1987). Following this martial analogy, it is worth noticing that the situation analyzed here can be described as one in which the incumbent is in the condition to use *secret traps* rather than visible barriers, in order to affect the scale and the occurrence of entry. In showing that commitment effects of unobservable investments can emerge in association to limit pricing, and secret traps mined by the incumbent can make entry unprofitable, our investigation is obviously related to the large literature on entry deterrence through public irreversible investments barriers (for a survey see Tirole (1986) and Ordover and Saloner (1989)).

The analysis of the benchmark model with observable investment and private information is more strictly related to contributions in Milgrom and Roberts (1986), Bagwell and Ramey (1988)[3] and, more recently in Bagwell (2007). A broad interpretation of observable investment as effort in promoting sales, i.e. advertising with a permanent effect on the installed base of customers as in Milgrom and Roberts (1986) and Bagwell (2007), is consistent with the basic features of our model.

The problem of the commitment value of choices observed with noise is analyzed in Bagwell (1995) and in Maggi (1999). Bagwell (1995) shows that, in a leader follower relationship, imperfectly observable actions do not have any commitment value. Maggi (1999) qualified this result by showing that imperfect observability of a choice does not necessarily destroy its commitment value when the first mover has

[3] In this latter model advertising has no long run commitment value in that the level of sales in the first period cannot directly affect sales in the second period.

private information of a specific type, that allows him to abstract from signaling considerations[4]. Although there are similarities, our analysis pertains to a different situation. Since the entrant's choice depends both on the expected type and on the expected long run investment, information transmission about both emerges from endogenous incentives in our case.

Our model with secret investment is also related to the analysis of a class of signaling models discussed by In and Wright (2012) where, however, there is no type heterogeneity and only signaling private choices is considered. There, an extensive discussion of the relevance of this kind of games in recent contributions in topics unrelated to entry deterrence can be found. A similar problem is also studied in Brighi, D'Amato and Piccolo (2005), albeit with a continuum of types for the incumbent and the entrant. The focus there is on the separating equilibrium only.

The rest of the paper is organized as follows: section 2 presents the basic model of entry in a monopolistic market and introduces the main assumptions; section 3 examines the benchmark entry deterrence model with observable investment; section 4 provides the results for the case of unobservable investments and compares the results. Finally, some remarks are offered in the concluding section. The proofs are collected in the appendix.

2 The entry model

We consider a standard two periods entry model where an incumbent firm faces the potential entry of a competing firm in a market for a homogeneous good. In the first period firm 1, the incumbent, who has private information about his costs of production, decides how much to produce, q, and how much to invest, e, in a cost reducing technology. In the second period firm 2, the entrant, after observing some or all of the incumbent's choices, decides whether to enter into the market. The entrant's choice is denoted by $y \in \{0, 1\}$, with $y = 1$ if entry takes place and 0

[4]The probability of different signals depends on the leader's choice, but the leader's type is immaterial, in other words incentive compatibility constraints for different types of leaders are not considered.

otherwise. If entry occurs the two firms compete, the entrant pays an entry cost, learns the incumbent's production costs (learning upon entry) and firms compete à la Cournot, otherwise firm 1 remains a monopolist and the entrant gains her outside option, normalized to zero.

In each period, the market behaviour is described by an inverse demand function $p(q)$, where $q \geq 0$ is the quantity of the homogeneous good. The function $p(q)$ is supposed to be differentiable and strictly decreasing. The marginal costs of firm 1 and firm 2 are constant and the fixed costs of production are set to zero, for convenience. The incumbent's marginal cost may take on different values and may also depend on the level of a long run investment in a cost reducing technology. Investment is made by the incumbent in the first period and it affects marginal costs in both periods. To simplify the analysis, we assume that there are only two types of incumbents, the L type with a low cost and the H type with a high cost, and denote by $\theta_t(e)$, with $t = L, H$, the marginal cost of type t when the amount of investment is $e \geq 0$. The cost reducing technology $\theta_t(e)$ is represented by a differentiable function of e, with $\theta_t(e) \geq 0$, and satisfies the following assumptions:[5]

A.1 $\theta_L(e) = \theta_L$ and $\theta_L < \theta_H(e)$ for all $e \geq 0$.

A.2 $\theta'_H(e) < 0$ for all e and $\theta''_H(e) > 0$ for all e above some threshold, $\widetilde{e} \geq 0$.

Assumption A.1 states that the low costs incumbent is on the technological frontier and hence no additional benefits can be obtained from investment. Also, A.1 states that investment may allow the high cost incumbent to reduce the cost gap with respect to type L, even though it will never let the ranking of types, in terms of marginal costs, be overturned. Assumption A.2 states that the high cost incumbent has strictly decreasing marginal costs and that the cost reducing technology exhibits decreasing returns to scale, at least for levels of investments above some given threshold. Two examples of the H type cost technologies consistent with A.1 and A.2 are depicted in Figure 1. The cost reducing technology $\theta^1_H(e)$ exhibits decreasing returns to scale and is more effective than $\theta^2_H(e)$, since it reaches lower unit costs with less investment. The cost technology $\theta^2_H(e)$ exhibits initial increasing returns and then decreasing returns

[5] For ease of notation we set $\theta_t = \theta_t(0)$.

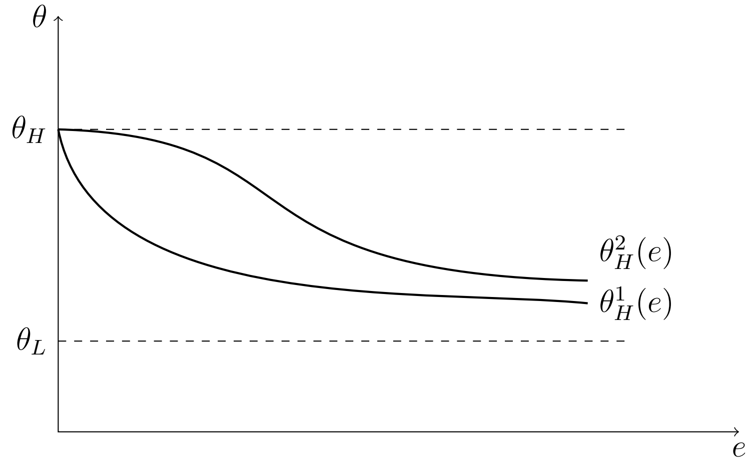

Figure 1: Two examples of cost reducing technology

to scale. None of these technologies allow the high cost incumbent to reach the unit cost of the H type.

The prior probability that the incumbent has high costs is denoted by β. The cost technology $\theta_H(e)$, the marginal cost θ_L and the prior probability β are common knowledge, as well as market demand $p(q)$, while the type t and the amount of investment e are incumbent's private information.

The single period incumbent's profits (gross of investment costs e) are given by

$$\Pi_t(e, q) = [p(q) - \theta_t(e)]q.$$

To simplify the analysis, let us assume that the profit function $\Pi_t(e, q)$ is *strictly quasiconcave* in q, so that, for each level of e, the incumbent's single period profit maximization problem has a unique solution given by the monopoly quantity, $m_t(e)$. The single period monopoly profit is $M_t(e) = \Pi_t(e, m_t(e))$.[6] It may be noticed that, by A.2, the high cost incumbent's functions $m_H(e)$ and $M_H(e)$ are continuous and strictly increasing.

The second period incumbent's profits depend on firm 2 entry decision. If firm 2 does not enter, the incumbent remains a monopolist and earns[7] $M_t(e)$, while firm

[6]For convenience of notation we set $m_t = m_t(0)$ and $M_t = M_t(0)$.

[7]The notation emphasizes that the incumbent's profits in the second period are a function of the

2 makes zero profits. If entry occurs, the entrant pays an 'entry fee', learns private information about incumbent's costs, i.e. the type t and the amount of investment e, and the two firms compete *à la Cournot*. The equilibrium duopoly profits of the incumbent depend on first period investments and are denoted by $D_t(e)$. The duopoly profits of the entrant, net of the entry fee, depend on incumbent's type and investment and are denoted by $D_{2t}(e)$.[8] For convenience of notation let us set $D_t = D_t(0)$ and $D_{2t} = D_{2t}(0)$.

The incumbent's decision about q and e, is based on *total profits* over the two periods. Total profits include first period investment expenditures and, assuming no time discounting, are given by

$$V_t(e, q, y) = \Pi_t(e, q) - e + yD_t(e) + (1 - y)M_t(e). \tag{1}$$

The entry decision of firm 2 depends on second period expected profits. After observing first period market quantity or equivalently the first period market price, the entrant makes an inference about incumbent's cost, i.e. on type and investment. Let $\hat{\beta}$ denote the entrant's *beliefs* about the probability of the incumbent being of type H and $\hat{e}$ the conjectures about incumbent's investment choice. The entrant expected profits in the case of entry are given by

$$\hat{\beta}D_{2H}(\hat{e}) + (1 - \hat{\beta})D_{2L}, \tag{2}$$

thus firm 2 enters if the expected profits are strictly positive. We make the following assumption of the entrant's duopoly profits.

A.3 $D_{2H} > 0$ and $D_{2L} < 0$; moreover $\beta D_{2H} + (1 - \beta)D_{2L} > 0$.

The first part of A.3 is a standard assumption in entry games and states that the entrant duopoly profits are strictly positive when the incumbent has high costs and are strictly negative otherwise. Therefore, entry is profitable unless the incumbent has low costs. The second part of A.3 states that the entrant expected profits at prior

first period long run investment choice.

[8] Clearly, the duopoly profits depend on a number of other factors, including marginal costs of the potential entrant and fixed entry costs, which are not explicitly parameterized.

beliefs, β, and in the absence of investment are positive. It rules out the possibility that the incumbent avoids entry without making any investment.

The next assumption specifies how the investments made by the incumbent in the first period affect the profitability of entry.

A.4 $D_{2H}(e) > 0$ for all $e \geq 0$. Moreover, there exists a level of investment, the *zero expected profit investment* denoted by $e_0 > 0$, such that the entrant's expected profit at prior beliefs is zero, i.e.

$$\beta D_{2H}(e_0) + (1 - \beta) D_{2L} = 0. \tag{3}$$

According to the first part of A.4, entrant's profits are always positive, when she faces a high cost incumbent, regardless of the level of investment undertaken. This assumption significantly limits the anticompetitive effect of investment, which, under complete and perfect information, would not enable the high cost incumbent to raise any entry barrier. However, the second part of A.4 ensures that there exists a level of investment making entry unprofitable in expected terms. This is the minimal assumption to let secret investment possibly support entry deterrence.

The entry model outlined above represents a sequential game with incomplete information and, specifically, a signaling game, where the presence of unobservable investments introduces into the analysis two new and interesting features. First, by reducing the marginal cost in the first period, the choice to invest by the H type lowers the cost of signaling, which becomes partially endogenous. This is one of the key elements of our model since, as we shall see, a greater incentive to mimic by the H type will also modify the incentive to signal by the L type. Second, because of the unobservable character of investment, the information carried by the pricing policy signal is two dimensional. Pricing policy may signal either the type, an exogenously given private characteristic, or the amount of investment, an endogenous unobservable action, or both. As we shall see, which dimension of the information will prevail in the signal depends on different equilibrium outcomes of the game.

The concept used to solve the signaling game is the standard notion of Perfect Bayes Nash equilibrium with an additional consistency requirement on the expectation about the secret investment. The intuitive criterion of Cho and Kreps (1987) is used to refine the multiplicity of equilibria, which is a typical characteristic of signaling games. For simplicity we only consider equilibria in pure strategies. As a benchmark for comparing later results, a signaling model with *publicly observable* investment will be analyzed in the next section. The analysis of the model with secret or *unobservable* investment is postponed to Section 4.

Finally, since general conditions for the existence of an equilibrium in separating strategies is not the scope of this paper, we shall make the following standard assumption[9]

$$M_L - D_L \geq M_H(e) - D_H(e), \qquad \text{for all } e, \tag{4}$$

which simply states that the low cost incumbent has greater benefits from entry deterrence, i.e. a greater difference between monopoly and duopoly profits, as compared to the high cost incumbent.

3 Observable investment

A relevant benchmark for our model is the case when investment by the incumbent is observable by the entrant. In this case investment plays a double role as a signal and as a commitment variable. In fact, long run investment recovers its commitment character because it is an irreversible and publicly observable choice affecting post entry competition. However, notice that, by assumption A.3, the role of investment as a commitment variable is limited because investment would not allow the incumbent to deter entry under complete information.

On the other hand, investment also acts as a signal, as does the quantity. As it is well known, quantity acts as a signal because producing a further quantity of the good is cheaper for the low cost incumbent or, more formally, because the quantity satisfies the 'single crossing' condition. Similarly, since by assumption A.1 investment is purely dissipative for type L and cost reducing for type H, making a zero level of investment is 'more expensive' for a high cost incumbent and may be interpreted by

[9] See, for instance, Tirole (1988) or Bagwell and Ramey (1988).

the entrant as a credible signal of a low cost type.[10] Therefore, the benchmark model is actually a special case of a game with multiple signals and in fact, it has some similarities with the signaling model with limit pricing and advertising studied by Bagwell and Ramey (1987).

Let us first describe the strategies of the signaling game and the notions of equilibrium applied to derive the solution. A pure strategy for the incumbent is a function which associates with each type a level of investment and a first period quantity and consists of two pairs, (e^H, q^H) and (e^L, q^L). After observing incumbent's choice, the potential entrant makes an inference about incumbent's costs. A *system of beliefs* for firm 2 is a function $\hat{\beta}$ which associate with any observable choice of the incumbent, (e, q), the ex post probability of the H type. A pure strategy of firm 2 associates with any observable choice of the incumbent the decision of whether to enter or not and is denoted by $y(e, q) \in \{0, 1\}$. The incumbent's payoff are given by (1) and the entrant's payoff by (2). A solution to the entry game is a *Perfect Bayesian Equilibrium* (PBE).

Definition 1. *A profile of strategies (e^t, q^t) and $y(e, q)$, with $t = H, L$, is a PBE of the signaling game with observable investment, if there exists a system of beliefs $\hat{\beta}(e, q)$ satisfying the following conditions:*

1. *The incumbent's strategy is optimal, i.e. for $t = H, L$*

$$(e^t, q^t) = \text{argmax } V_t(e, q, y(e, q)).$$

2. *The entrant's strategy is optimal, i.e. $y(e, q) = 1$ if and only if*

$$\hat{\beta}(e, q)D_{2H}(e) + (1 - \hat{\beta}(e, q))D_{2L}(e) > 0.$$

3. *The beliefs $\hat{\beta}(e, q)$ are consistent with Bayes rule whenever possible.*[11]

Signaling games usually have a multiplicity of equilibria arising from the lack of restrictions on off-equilibrium beliefs. To deal with multiplicity we apply the intuitive

[10] Notice that investment retains its character of signal even when assumption A.1 is relaxed by allowing the investment to be cost reducing for the L type but not as much as for the H type.

[11] Specifically, if $(e^H, q^H) = (e^L, q^L)$ then $\hat{\beta}(e^H, q^H) = \beta$ and if $(e^H, q^H) \neq (e^L, q^L)$ then $\hat{\beta}(e^H, q^H) = 1$ and $\hat{\beta}(e^L, q^L) = 0$.

criterion originally proposed by Cho and Kreps (1987) and based on the following notion of dominated choice. Let $\hat{V}_t$ denote the payoff of type t in a given equilibrium. A deviation from the equilibrium choice, $(\tilde{e}, \tilde{q}) \neq (e^t, q^t)$, is *equilibrium dominated for type* t if, when entry does not take place, type t is worse off than in equilibrium, i.e. if $V_t(\tilde{e}, \tilde{q}, 0) < \hat{V}_t$. A system of beliefs supporting a given equilibrium satisfies the *intuitive criterion* if whenever a deviation is equilibrium dominated for type t and strictly preferred by type t', the entrant assigns the deviation to type t'. For example, if the deviation $(\tilde{e}, \tilde{q})$ is equilibrium dominated for H and strictly preferred by L, i.e.

$$V_H(\tilde{e}, \tilde{q}, 0) \;<\; V_H(e^H, q^H, y(e^H, q^H)) \tag{5}$$

$$V_L(\tilde{e}, \tilde{q}, 0) \;>\; V_L(e^L, q^L, y(e^L, q^L)), \tag{6}$$

the off-equilibrium belief must be $\hat{\beta}(\tilde{e}, \tilde{q}) = 0$. An *intuitive equilibrium* is a PBE which can be supported by a system of beliefs satisfying the intuitive criterion. It turns out that, in the present setting, intuitive equilibria can be characterized as follows.[12]

Fact 1. *A PBE, (e^t, q^t) and $y(e, q)$, with $t = H, L$, is intuitive if and only if there exists no deviation $(\tilde{e}, \tilde{q}) \neq (e^t, q^t)$ such that (5) and (6) hold.*

The next two subsections deal with two kinds of PBE, separating equilibria and pooling equilibria.

3.1 Separating equilibrium

A separating equilibrium is a Perfect Bayesian Equilibrium where different types of the incumbent firm make different choices or, equivalently, where $(e^H, q^H) \neq (e^L, q^L)$. In a separating equilibrium information is fully revealed and hence, by assumption A.3, the entrant only enters when she believes she is facing a high cost incumbent. The H type accommodates entry in the second period, i.e. he produces the monopoly quantity in the first period and the duopoly quantity after entry, and decides a level

[12] This characterization is also used in Bagwell and Ramey (1988). A sketch of the proof can be found in Appendix 1.

of investment which maximizes total profits. The equilibrium choice for the H type is $e^H = e_A$ and $q^H = m_H(e_A)$, where e_A is the level of investment maximizing total profits, i.e. the solution to the problem

$$\max_e V_H(e, m_H(e), 1). \tag{7}$$

The type H maximum total profit of accommodation is given by

$$V_H^A = M_H(e_A) - e_A + D_H(e_A). \tag{8}$$

The equilibrium choice of the L type, (e^L, q^L), must satisfy the 'incentive compatibility condition' of the H type, i.e.

$$V_H(e^L, q^L, 0) \leq V_H^A. \tag{9}$$

In other words, the high cost incumbent should prefer accommodating entry rather than playing (e^L, q^L). Furthermore, the equilibrium choice of the L type must provide him with total profits which are greater than the 'accommodation profit', i.e. the total profit he may earn by making zero investment, playing the monopoly quantity in the first period and by playing the duopoly quantity in the second, given entry. The type L total profit of accommodation is given by

$$V_L^A = V_L(0, m_L, 1) = M_L + D_L \tag{10}$$

whereas the equilibrium choice (e^L, q^L) must satisfy the 'participation condition'

$$V_L(e^L, q^L, 0) \geq V_L^A, \tag{11}$$

which is equivalent to $\Pi_L(q^L) - e^L \geq D_L$.[13]

In order to avoid the trivial case where the L type separates by simply choosing its monopoly quantity, we must assume that the high cost incumbent has an incentive to mimic the low cost one, i.e. that type H prefers to behave like type L, if this allows him to avoid entry. Therefore, we assume that the following 'mimicking condition' holds throughout the paper:

$$V_H(0, m_L, 0) > V_H^A. \tag{12}$$

[13]Since the L type profit does not depend on e, for ease of notation, we set $\Pi_L(q) = \Pi_L(e, q)$.

It is not difficult to see that an incumbent strategy with $(e^H, q^H) = (e_A, m_H(e_A))$ and (e^L, q^L) satisfying (9) and (11) supports a separating equilibrium. Indeed, one can take the system of beliefs which assign to the high cost type any choice different from (e^L, q^L), i.e. $\hat{\beta}(e, q) = 1$ for any $(e, q) \neq (e^L, q^L)$ and 0 otherwise, and the entrant strategy $y(e, q) = 1$ for any $(e, q) \neq (e^L, q^L)$ and 0 otherwise. This profile of strategies and the given system of beliefs satisfy Definition 1.[14]

As it is well known, the freedom in choosing off equilibrium beliefs gives rise to a multiplicity of equilibria. Indeed, the L type can be induced to choose any pair (e^L, q^L) satisfying (9) and (11), by choosing *ad hoc* off equilibrium beliefs which trigger entry in the case a deviation is observed. To refine the set of equilibria we apply the intuitive criterion by using Fact 1. It turns out that there is a unique intuitive separating equilibrium where the low cost incumbent is able to signal himself by producing in excess of the monopoly level and setting investment to zero. In order to formally prove the above statement, let us derive an intermediate result which characterizes incumbent strategies supporting intuitive separating equilibria.

Lemma 1. *An incumbent strategy, (e^t, q^t) with $t = H, L$, supports an intuitive separating equilibrium if and only if $(e^H, q^H) = (e_A, m_H(e_A))$ and (e^L, q^L) is a solution to the following maximization problem*

$$\max_{e,q} \quad \Pi_L(q) - e$$

$$\textit{subject to} \quad \Pi_H(e, q) - e + M_H(e) \leq V_H^A \tag{13}$$

$$\Pi_L(q) - e \geq D_L \tag{14}$$

The proof of Lemma 1 is in Appendix 1. It can be noticed that, if the H type has no incentive to mimic type L, i.e. if (12) does not hold, then $(0, m_L)$ trivially satisfies the constraint (13) (and (14)) and maximizes the single period profit in Lemma 1.

[14] Indeed, $\hat{\beta}(e, q)$ fulfils Bayes rule, given the profile of strategies. The entrant strategy is optimal since expected profits are negative only at (e^L, q^L) and $y(e, q) = 0$ only at $(e, q) = (e^L, q^L)$. The H type choice is optimal because of (7) and (9); the L type choice is optimal since any deviation from (e^L, q^L) triggers the entry of firm 2 and lowers incumbent profits.

As a result, $(e^L, q^L) = (0, m_L)$ supports the unique intuitive separating equilibrium.[15] A unique intuitive separating equilibrium also exists under assumption (12) and is characterized by the following proposition, which is proved in Appendix 1.

Proposition 1. *In the entry model with observable investment, there exists a unique separating equilibrium satisfying the intuitive criterion. The equilibrium is supported by the incumbent strategy* $(e^H, q^H) = (e_A, m_H(e_A))$ *and* $(e^L, q^L) = (0, q^*)$*, where* q^* *is implicitly defined by the equation*

$$V_H(0, q^*, 0) = V_H^A \tag{15}$$

and $q^* > m_L$.

Due to the mimicking condition (12), the intuitive separating equilibrium quantity q^* will exceed the level of monopoly, as depicted in Figure 2. In equilibrium, the high cost incumbent chooses the optimal level of investment accommodating entry, which is the same as under complete information, and fixes his monopoly price. The low cost incumbent makes zero investment, he chooses a price below his monopoly price and he keeps the entrant out of the industry. The limit price, however, does not deter entry, because entry does not take place exactly when it is unprofitable.

3.2 Pooling equilibrium

A pooling equilibrium is a PBE where all types of incumbent send the same signals or, equivalently, where $(e^L, q^L) = (e^H, q^H) = (e^P, q^P)$. Since the entrant does not learn any new piece of information from the observation of the equilibrium choice, the beliefs about the incumbent type are unmodified and are equal to the prior probabilities, i.e. $\hat{\beta}(e^P, q^P) = \beta$. If the entrant's expected profits at prior beliefs were positive, a pooling strategy would fail to deter entry of firm 2, therefore a pooling strategy could not be an equilibrium, because any type of incumbent would rather

[15] By uniqueness we mean that the same incumbent strategy is shared by all the intuitive separating equilibria.

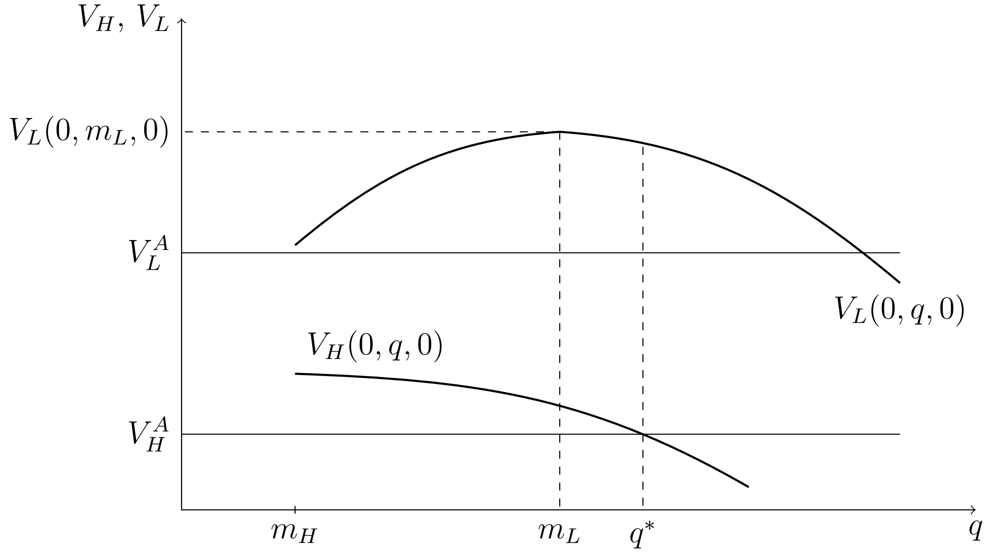

Figure 2: Intuitive separating equilibrium quantity.

choose his monopoly quantity in the first period and then accommodate entry. A necessary condition for a pooling equilibrium is that, at prior beliefs, the expected profits of the entrant are not strictly positive. Assumptions A.4 and A.5 imply that a pooling equilibrium can only exist when the incumbent makes positive investments. Specifically, the equilibrium level of investment can not be lower than the zero expected profit level defined by (3), otherwise firm 2 would not be induced to stay out, thus $e^P \geq e_0$. Furthermore, in a pooling equilibrium, the incumbent strategy, (e^P, q^P), must allow each type to obtain at least his respective 'reservation payoff', which means that the following 'participation conditions' must hold:

$$V_H(e^P, q^P, 0) \geq V_H^A \tag{16}$$

$$V_L(e^P, q^P, 0) \geq V_L^A \tag{17}$$

The following lemma, which is proved in Appendix 1, provides a characterization of pooling equilibria.

Lemma 2. *The incumbent strategy $(e^t, q^t) = (e^P, q^P)$, with $t = H, L$, supports a pooling equilibrium if and only if (e^P, q^P) satisfies (16), (17) and $e^P \geq e_0$, where e_0*

is the zero expected profit level of investment defined by (3).

The existence of a pooling equilibrium is not guaranteed in the model with observable investment. Indeed, e_0 may be so high that none of the incumbent types would find it worthwhile to invest. For instance, if e_0 was greater than the maximum gain from entry deterrence, $M_L - D_L$, the L type had no incentive to play the pooling strategy. Similarly, it would not be worthwhile for type H to invest if the savings in the costs of production were less than the expenditure e_0. In such cases no pooling equilibrium exists.

Conditions ensuring the existence of a pooling equilibrium require that the gains from entry deterrence for both incumbent types exceed investment expenditures e_0. An example of sufficient conditions is $M_L - D_L \geq e_0$ and $V_H(e_0, m_L, 0) \geq V_H^A$. Under the above conditions there is a multiplicity of pooling equilibria and, specifically, there exists a pooling equilibrium supported by the incumbent strategy $(e^P, q^P) = (e_0, m_L)$, i.e. the equilibrium where both types play the minimum level of investment achieving entry deterrence (zero expected profits to the entrant) and the monopoly quantity of the low cost incumbent.[16]

It is interesting to notice that, in a pooling equilibrium, investment has a role as a commitment variable vis à vis the entry decision. It is true that both e^P and q^P act as signals, although they provide no new piece of information about the cost type of the incumbent. It is the observed level of investment above e_0 which modifies the perceived profitability of entry of firm 2 and thus deters entry. Matched with pricing policy, investment gains quite a strong commitment value when compared to equilibrium investment arising in the model with complete information on θ_i. In order to be supported, however, the pooling equilibrium requires type L to send a completely dissipative signal in terms of investment expenditures. Is this dissipation plausible?

Provided that a pooling equilibrium exists, in order to be a plausible outcome of the game it must satisfy the intuitive criterion. By Fact 1, a pooling equilibrium supported by the incumbent strategy (e^P, q^P) is intuitive if there exists no deviation

[16]This claim is easily proved by using Lemma 2.

$(\tilde{e}, \tilde{q})$ such that

$$\begin{aligned} V_H(\tilde{e}, \tilde{q}, 0) &< V_H(e^P, q^P, 0) \\ V_L(\tilde{e}, \tilde{q}, 0) &> V_L(e^P, q^P, 0) \end{aligned}$$

Indeed, if such a deviation existed then (e^P, q^P) could be an optimal choice for type L only if $\hat{\beta}(\tilde{e}, \tilde{q}) = 1$, which is an unintuitive belief. It turns out that no pooling equilibrium passes the test of the intuitive criterion.

Proposition 2. *In the entry game with observable investment there are no pooling equilibria satisfying the intuitive criterion.*

The proof of Proposition 2 is in Appendix 1. The purely dissipative nature of investment expenses by type L explains the collapse of the pooling under the intuitive criterion. By cutting down investment to zero and increasing quantities above the level stipulated in the candidate pooling equilibrium, an incumbent of type L can convince the entrant that such a low price in the deviation with no investment can not be profitable by incumbents of types H, whereas it allows type L to save on the dissipation of useless expenses. More generally, it is the fact that investment has a different impact across types, and so can act as a signal, to make the pooling equilibrium impossible to survive the application of the intuitive criterion in the benchmark model.[17]

The main conclusion of section 3 follows from Proposition 1 and 2. The analysis of the model with observable investment provides an unambiguous prediction of the outcome of the game, which is the unique intuitive separating equilibrium characterized in Proposition 1. The high cost incumbent accommodates entry, while the low cost incumbent leaves the potential entrant out of the industry by playing a price below the monopoly level. Limit pricing, however, does not limit entry, since entry does not take place exactly when it is unprofitable.

[17]It is interesting to notice that the intuitive criterion does not rule out, in general, pooling equilibria either in the original formulation by Milgrom and Roberts (1982) or in the model with advertising by Bagwell and Ramey (1987).

4 Unobservable investment

This section deals with the case when long run investment is not observable by the entrant. Since investment is not publicly observable it can not act directly either as a signal or as a commitment variable for entry deterrence objectives. However investment may affect the cost of signaling and, provided some information about the level undertaken by the incumbent is contained in the pricing strategy, a commitment value for entry deterrence purposes can emerge in equilibrium.

There are two main differences with respect to the benchmark model of section 3, which are worth noticing. First, there is a piece of information which is no more available to the entrant. Without any information about investment, the entrant can not determine the cost of signaling of the high cost incumbent and in addition, she would not be able to make an ex ante assessment of entry profitability. Therefore, the entrant now strives for making inferences not only about the incumbent's type, but also about the incumbent's action. Second, the number of signals passes from two to one. The price is the only signal left which may allow the entrant to receive information relevant for her entry decision. Pricing policy may now convey information not only about cost type, but also about the level of investment undertaken by the high cost incumbent. Hence, the high cost incumbent may have incentives to convey information about the level of investment undertaken being enough to make entry unprofitable. The analysis of the present section, indeed, will show that the solution to the entry problem depends on the information content that the price signal is able to convey in equilibrium.

Let us start the analysis by describing strategies and solution concepts of our model. Incumbent strategies are the same as in section 3, i.e. (e^t, q^t) with $t = H, L$, although the choice of e is not publicly observable. The entrant only observes the choice of q and makes inferences about the cost type and about the level of investment undertaken by the high cost incumbent.[18] Formally, we denote by $\hat{\beta}(q)$ the belief, i.e. the posterior probability that the incumbent has high costs, and by $\hat{e}(q)$ the

[18]Since zero investment is a dominant choice for the low cost incumbent, the inference about the L type investment is trivially zero.

conjecture on investment, i.e. the estimate of the level of the H type investment made by the entrant after the quantity q is observed. The entrant strategy is a function associating a binary entry decision with first period quantities, i.e. $y(q) \in \{0, 1\}$. The incumbent's payoff is given by (1). The entrant's payoff in the case of entry is given by her expected profits, i.e. $\hat{\beta}(q)D_{2H}(\hat{e}(q)) + (1 - \hat{\beta}(q))D_{2L}$, and is zero otherwise. The definition of Perfect Bayesian equilibrium is modified as follows.

Definition 2. *A profile of strategies* (e^t, q^t) *and* $y(q)$*, with* $t = H, L$*, is a PBE of the signaling game with unobservable investment if there exists a system of beliefs* $\hat{\beta}(q)$ *and a system of conjectures* $\hat{e}(q)$*, such that the following conditions are satisfied:*

1. *The incumbent's strategy is optimal, i.e. for* $t = H, L$

$$(e^t, q^t) = \text{argmax } V_t(e, q, y(q))$$

2. *The entrant's strategy is optimal, i.e.* $y(q) = 1$ *if and only if*

$$\hat{\beta}(q)D_{2H}(\hat{e}(q)) + (1 - \hat{\beta}(q))D_{2L} > 0$$

3. *The beliefs are consistent with Bayes rule, whenever possible, and the conjectures on investment are consistent with the H type choice, i.e.* $\hat{e}(q^H) = e^H$.

We notice that, by Definition 2.3, the entrant's inference on type and investment must be consistent with the incumbent's strategy in equilibrium. Beliefs and investment conjectures off the equilibrium path are not restricted, however, and this freedom of choice is the source of the multiplicity of equilibria.

A tailored version of the intuitive criterion will be applied to deal with the multiplicity of equilibria in our signaling model. To carry out the analysis, however, we have to specify the way investment conjectures are made by the entrant and this requires a closer look at the optimizing choices of the high cost incumbent.

To facilitate the analysis of our model we shall introduce two auxiliary mathematical tools that will be used throughout this section. Let us define a hypothetical investment function which provides the optimal level of investment chosen by the high cost incumbent at a given quantity and under the hypothesis that entry does not take place. The *investment function*, denoted by $\phi(q)$, is obtained as the solution to the problem of maximizing the H type total profit function at q and given no entry, i.e.

$$\phi(q) = \text{argmax}_e\, V_H(e, q, 0) = \Pi_H(e, q) - e + M_H(e). \tag{18}$$

Under our assumptions on technology, the maximization problem defining the investment function has a unique solution for any $q \in [m_H, q_c]$, where q_c is the competitive market quantity. The *(maximum) value function* of the maximization problem,

$$U(q) = V_H(\phi(q), q, 0), \tag{19}$$

provides the maximum total profit of type H at the quantity q under the hypothesis that entry does not take place and the incumbent chooses investment optimally. The functions $\phi(q)$ and $U(q)$ are well defined and continuous in the interval $[m_H, q_c]$; $\phi(q)$ is strictly increasing and $U(q)$ is a strictly quasi-concave function with a global maximum at $\bar{m}$, the monopoly quantity of an unthreatened H type monopolist.[19] The main characteristics of the value function are illustrated in Figure 3, where the graph of $U(q)$ is plotted against the graphs of the total profit functions of both types of incumbent in the absence of investment and without entry.

The Intuitive Criterion is modified to fit our model by using the functions defined by (18) and (19). In facing a deviation from a candidate equilibrium the entrant is forced to think both about which type originated the deviation *and* what level of investment corresponds to that deviation. By Definition 2 it is clear that, if the deviation is ascribed to type L, then the associated investment behind that deviation must be zero. If the deviation is ascribed to type H, then the associated investment must be given by $\hat{e}(q) = \phi(q)$. Accordingly, the value function $U(q)$ will provide the highest payoff that the H type incumbent may obtain from the deviation.

The modified version of the Intuitive Criterion is a straightforward adaptation of the definition given in section 3 using the above assumption on investment conjectures. Let $\hat{V}_t$ denote the payoff of type t in a given equilibrium. A deviation from

[19] In the absence of any entry threat, the high cost incumbent chooses a level of investment which maximizes total profits, $V_H(e, m_H(e), 0)$. If investment is profitable, the solution is a strictly positive level of investment, $\bar{e}$, and the monopoly quantity is $\bar{m} = m_H(\bar{e})$. The properties of $\phi(q)$ and $U(q)$ are proved in Appendix 2, Lemma A2.0.

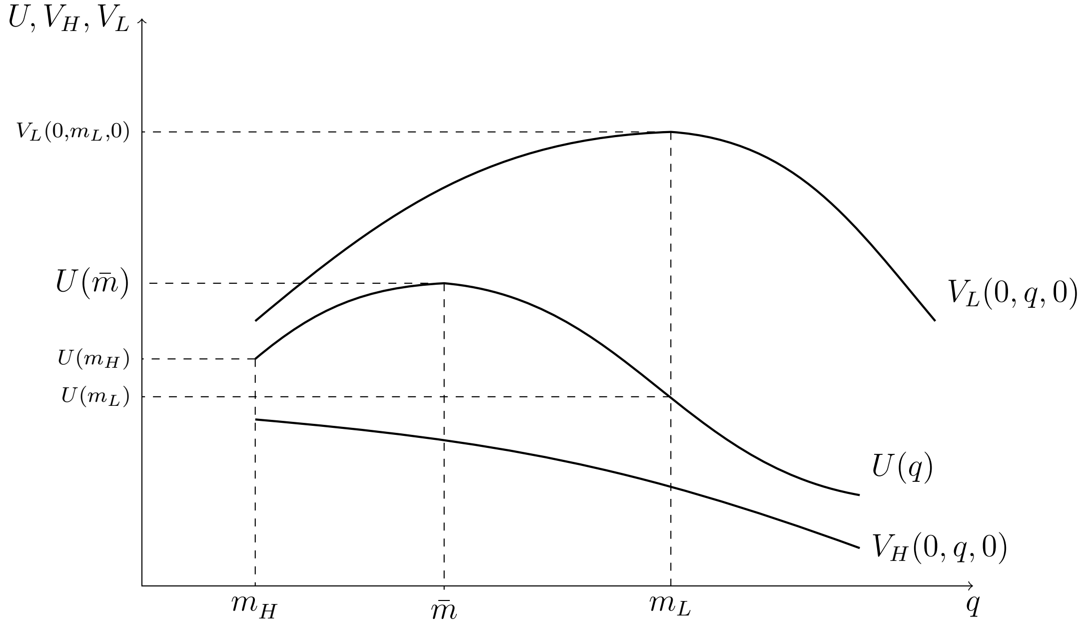

Figure 3: Total profits V_H, V_L and the value function $U(q)$

the equilibrium choice, $\tilde{q} \neq q^t$, is *equilibrium dominated for type* t if, when entry does not take place and the incumbent makes his best investment choice given $\tilde{q}$, type t is worse off than in equilibrium. The main difference with respect to section 3 is that the total profit from deviation of the H type is the maximum profit given by the value function $U(\tilde{q}) = V_H(\phi(\tilde{q}), \tilde{q}, 0)$. The definition of the Intuitive Criterion is the same as in section 3, with the modified notion of equilibrium domination, and the intuitive equilibria can be characterized by a simple condition.

Fact 2. *A PBE,* (e^t, q^t) *and* $y(q)$*, with* $t = H, L$*, satisfies the intuitive criterion if and only if there exists no deviation* $\tilde{q} \neq q^t$ *such that*

$$\begin{aligned} U(\tilde{q}) &< V_H(e^H, q^H, y(q^H)) \quad \textit{and} \\ V_L(0, \tilde{q}, 0) &> V_L(0, q^L, y(q^L)). \end{aligned}$$

As in Fact 1, the Intuitive Criterion requires that, under the best entry conditions that the deviating firm may face, there is no deviation which is equilibrium dominated for the H type and strictly preferred by the L type to the equilibrium choice.

The two cases of pure strategies PBE, namely separating equilibria and pooling equilibria, will be analysed in the following subsections.

4.1 Separating equilibrium

In a separating equilibrium different types choose different quantities and information is revealed to the entrant. Therefore, Firm 2 enters when the incumbent has high costs and remains out when Firm 1 is of type L. The H type will choose a monopoly quantity in the first period and accommodate entry in the second. The choice of the H type in a separating equilibrium, thus, is the same as that seen in Section 3, i.e. $e^H = e_A$ and $q^H = m_H(e_A)$.

The high cost incumbent makes zero investment, because by assumption A.1, investment is purely dissipative. In the choice of the quantity q^L, the L type takes into account the maximum profits that the H type may earn if he chooses q^L and avoids entry, which is given by the value function $U(q^L) = V_H(\phi(q^L), q^L, 0)$. Therefore, the choice of q^L must satisfy an incentive compatibility condition given by

$$U(q^L) \leq V_H^A. \tag{20}$$

The inequality (20) states that the maximum profit at q^L of the high cost incumbent when entry does not take place, can not exceed his profits of accommodation. With respect to the analysis of section 3, the unobservability of investment makes the incentive compatibility condition more severe, because it allows the H type to choose investment optimally increasing profits at any given signal.

The choice of the quantity q^L in a separating equilibrium must also satisfy the participation condition of type L, that is

$$V_L(0, q^L, 0) \geq V_L^A. \tag{21}$$

Notice that (21) is equivalent to $\Pi_L(q^L) \geq D_L$.

In the model with unobservable investment, separating equilibria exist under standard conditions stated in (4) as it is shown below. The next result characterizes incumbent strategies supporting separating equilibria satisfying the intuitive criterion.

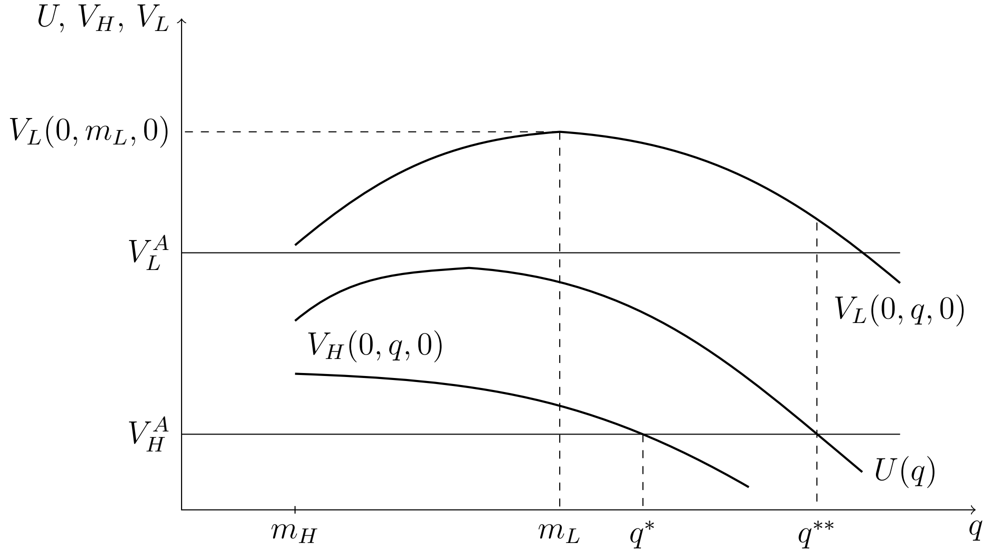

Figure 4: Intuitive separating equilibrium quantities q^* and q^{**}

Lemma 3. *The incumbent strategy $(e^H, q^H) = (e_A, m_A)$ and (e^L, q^L) supports an intuitive separating equilibrium if and only if $e^L = 0$ and the quantity q^L is a solution to the following maximization problem:*

$$\begin{aligned} \max_q \quad & \Pi_L(q) \\ \textit{subject to} \quad & U(q) \leq V_H^A \\ & \Pi_L(q) \geq D_L \end{aligned}$$

According to Lemma 3, whose proof is in Appendix 2, the separating quantity is characterized as the solution to the problem of maximizing first period profits of the low cost incumbent subject to the incentive compatibility constraint (20) and the participation constraint (21). By applying Lemma 3, it turns out that there exists a unique intuitive separating equilibrium.

Proposition 3. *In the entry model with unobservable investment, there exists a unique separating equilibrium satisfying the intuitive criterion. This equilibrium is supported by the incumbent strategy $(e^H, q^H) = (e_A, m_A)$ and $(e^L, q^L) = (0, q^{**})$,* *where q^{**} is implicitly defined by the equation*

$$U(q^{**}) = V_H^A. \tag{22}$$

Moreover, $q^{**} > q^*$, where q^* is the intuitive separating equilibrium quantity with observable investment, defined by (15).

The proof is in Appendix 2. In the unique intuitive separating equilibrium, the high cost incumbent accommodates entry, while the low cost one keeps the entrant out by producing a greater quantity as compared to the case where investment is observable. The equilibrium quantities under observable and under unobservable investment, respectively q^* and q^{**}, are shown in Figure 4.

The further distortion in the limit price has a simple economic interpretation. Secrecy allows now the high cost incumbent to invest without revealing any information about his cost type, reducing his cost of mimicking, as compared to the case of observability, i.e. increasing the signaling cost to type L. The entrant is aware that, now, any price signal may be cheaper for the H type, who has a stronger incentive to mimic the low cost incumbent. As a result the 'old' limit price cannot be a credible signal of the cost type and the entrant requires an even more distorted price signal to achieve separation. Clearly, the more effective is the cost reducing technology, the greater will be the distortion of the limit price in equilibrium.

Finally, as we also noticed in section 3, the limit price in a separating equilibrium does not deter entry, which takes place exactly when it is profitable.

4.2 Pooling equilibrium

In a pooling equilibrium, both types of incumbent choose the same quantity, q^P, and the potential entrant stays out of the market. The entrant does not learn anything about the type of incumbent, but her conjecture $\hat{e}(q^P)$ is that the high cost type has made at least the 'zero expected profit' level of investment e_0, i.e. $\hat{e}(q^P) \geq e_0$. Indeed, any other conjecture would induce firm 2 to enter according to assumption A.5. The investment choice of the H type in a pooling equilibrium, e^H, must be optimal and since entry does not take place, it must be given by the investment function, i.e. $e^H = \phi(q^P)$. By Definition 2.3, the entrant's conjecture must be correct, i.e. $\hat{e}(q^P) = e^H$, therefore the pooling equilibrium quantity must satisfy the condition $\phi(q^P) \geq e_0$. In other words, the entrant stays out of the market if she observes a quantity sufficiently large, and particularly, greater than the *deterrence quantity* q_0, implicitly defined by

$$\phi(q_0) = e_0. \tag{23}$$

Indeed, since $\phi(q)$ is strictly increasing, any quantity greater than q_0 reveals to the entrant that the high cost incumbent has made at least the zero expected profit level of investment.

The following result, which is proved in Appendix 2, characterizes the incumbent strategy in pooling equilibria.

Lemma 4. *An incumbent strategy* (e^H, q^H)*,* (e^L, q^L)*, with* $q^H = q^L = q^P$ *and* $e^L = 0$ *supports a pooling equilibrium if and only if*

(i) $U(q^P) \geq V_H^A$

(ii) $V_L(0, q^P, 0) \geq V_L^A$

(iii) $e^H = \phi(q^P)$ *and* $e^H \geq e_0$.

Conditions (i) and (ii) of Lemma 4 are the participation conditions of the two types of incumbent. The maximum profit function $U(q)$ in condition (i) is due to the fact that, under unobservability, the high cost incumbent in a pooling equilibrium can choose investment optimally given the signal and given no entry.

It may be noticed that, by condition (i) of Lemma 4, the equilibrium quantity cannot exceed the separating equilibrium level q^{**} defined by (22), otherwise the H type would make higher profits by accommodating entry. Condition (iii), on the other hand, sets a lower bound to q^P, which is the deterrence quantity q^0. Therefore, the equilibrium quantity is to be found between q_0 and q^{**}, and no pooling equilibrium can exist if the deterrence quantity is too high, i.e. if $q_0 > q^{**}$.

The next result shows that, if $q_0 < q^{**}$, there exist pooling equilibria satisfying the intuitive criterion and, specifically, there is a unique intuitive pooling equilibrium which is *Pareto undominated* according to the incumbent's interim payoff, i.e. such that none of the types of incumbent can be made better off in any other intuitive pooling equilibrium.

Proposition 4. *Let $q_0 < q^{**}$, where q_0 is the deterrence quantity defined by (23) and q^{**} is given by (22).*

(i) Any intuitive pooling equilibrium is supported by a quantity q^P such that $q_0 \leq q^P \leq q^{**}$, if $q_0 > m_L$, and $m_L \leq q^P \leq q^{**}$, if $q_0 \leq m_L$.

(ii) The quantities $q^P = q_0$, if $q_0 > m_L$, and $q^P = m_L$, if $q_0 \leq m_L$, support the unique intuitive pooling equilibrium which is Pareto undominated, by other intuitive pooling equilibria, according to the incumbent's interim payoff.

The proof is in Appendix 2.[20] Proposition 4 can be illustrated by means of Figure 5. Two cases are depicted, the case where the deterrence quantity is smaller than the low cost monopoly quantity, i.e. $q_0' < m_L$, and the the case where the deterrence quantity is larger, i.e. $q_0 > m_L$. In the first case, consider the pooling equilibrium supported by $q^P = q_0'$. The deviation $\tilde{q} = m_L$ is equilibrium dominated for type H, since $U(q_0') > U(m_L)$, and it is strictly preferred by type L since m_L maximizes his profits. Thus, according to Fact 2, the equilibrium supported by $q^P = q_0'$ does not satisfy the Intuitive Criterion. In the second case depicted in Figure 5, the deterrence quantity is $q_0 > m_L$. Take the pooling equilibrium supported by the quantity $q^P = q_0$. Any deviation $\tilde{q} < q_0$ is not equilibrium dominated for type H, since $U(q_0) < U(\tilde{q})$. Moreover, any deviation $\tilde{q} > q_0$ is equilibrium dominated for both types of incumbent. Therefore, $q^P = q_0$ satisfies the Intuitive Criterion according to Fact 2. Notice also that, when the deterrence quantity is q_0', it is easily checked on Figure 5 that the equilibrium supported by $q^P = m_L$ satisfies the Intuitive Criterion, as stated by

[20]The result for $q_0 > m_L$ requires an additional mild assumption stating that the low cost incumbent profits are greater for quantities close to m_L, rather than for quantities close to m_H. This assumption, together with the complete proof is in the appendix.

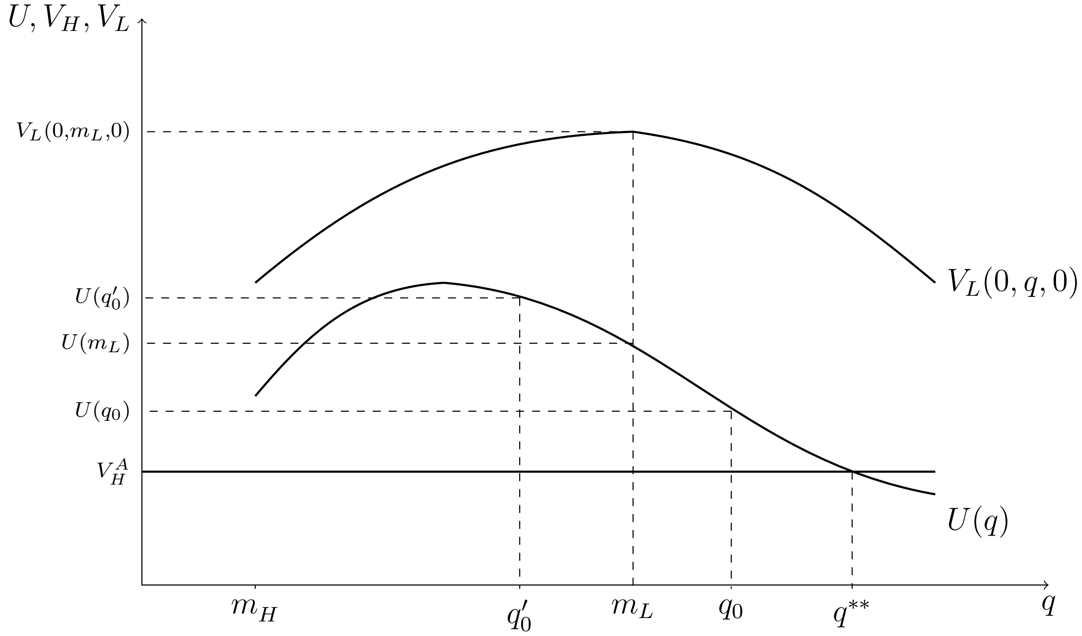

Figure 5: Intuitive pooling equilibria

Proposition 4. Finally, to see that the intuitive pooling equilibrium supported by $\hat{q}$ is Pareto undominated for the incumbent, it is sufficient to notice that both functions, $U(q)$ and $V_L(0, q, 0)$, are strictly decreasing for $q \geq m_L$ and that intuitive pooling equilibrium quantities must be greater than $\hat{q}$, which in turns is greater than m_L.

Proposition 3 and Proposition 4 provide us with two candidates for the solution to the entry problem when investment is unobservable, a separating equilibrium and a pooling equilibrium. Both equilibria are intuitive, but the pooling is Pareto superior to the separating equilibrium, from the point of view of the incumbent, i.e. both types of incumbent are better off in the pooling equilibrium.[21] Since the outcome of the intuitive pooling equilibrium characterized by Proposition 4 is strictly preferred by both types of incumbent, it seems the most plausible solution to the entry problem with unobservable investment.

[21] Indeed, since $\hat{q} < q^{**}$ we have $U(\hat{q}) > U(q^{**})$ and $V_L(0, \hat{q}, 0) > V_L(0, q^{**}, 0)$. This result can also be seen graphically by comparing incumbent's payoff at q^{**} and at q_0 in Figure 5.

The selection of the intuitive pooling equilibrium can also be justified on more formal grounds by resorting to the notion of 'defeated' equilibrium proposed by Mailath, Okuno-Fujiwara and Postlewaite (1993). This refinement places restrictions on off equilibrium beliefs and, roughly speaking, relies on the following idea. A deviation from an equilibrium, say E_1, should be interpreted as a the signal that a different equilibrium, say E_2, was played, therefore the beliefs of the deviation from E_1 should be the equilibrium belief of E_2. Let us apply this refinement to our model and, specifically, to the deviation $\tilde{q} = \hat{q}$ from the separating equilibrium quantity q^{**}. By the above argument, at the deviation $\tilde{=}\hat{q}$ should by assigned the belief of the pooling equilibrium quantity $\hat{q}$, which is the prior belief β. However, if the off equilibrium belief $\hat{\beta}(\tilde{q}) = \beta$ is assigned to the deviation $\tilde{q} = \hat{q}$, the optimal choice of firm 2 is to stay out of the market if $\hat{q}$ is observed, so that the choice of q^{**} by the L type is not optimal any more. Thus, the separating equilibrium collapses and it turns out to be a 'defeated' equilibrium. By applying the analysis of Mailath, Okuno-Fujiwara and Postlewaite (1993), it can also be shown that, the only 'undefeated' equilibrium in the model with unobservable investment is the intuitive pooling equilibrium characterized in Proposition 4.

In the presence of unobservable long run investments affecting the cost of signaling, we expect to observe entry deterrence through a limit price which is not as low as it would be if the price were to reveal with certainty that the incumbent has low costs. In equilibrium, the 'pooling limit price' does not convey any information about the cost type, but it conveys information about investment behaviour of the high cost incumbent. By observing the equilibrium price the potential entrant learns that the H type has made investments so as to make expected profits of firm 2 not positive and decides to stay out of the market. Therefore, we conclude that limit pricing is an effective strategy to deter entry, which does not occur even in those cases where entry is profitable.

5 Final remarks

In this paper we revisited what we think is an important aspect of the theory of limit pricing: if and how public short run decisions by incumbents are intertwined with long run irreversible secret investment choices made in order to affect the potential entrant's decision. To this aim we constructed a signaling game featuring a standard entry model *à la* Milgrom and Roberts (1982), with private information about costs, where the incumbent can attempt to forestall entry by his pricing policy and by investing in a cost reducing activity, which is not directly observable by a potential entrant. As in the standard model, it is assumed that entry is profitable for the potential entrant only in the presence of a high cost incumbent, regardless of the amount of investment undertaken. This does not rule out the existence of a level of investment by the high cost incumbent such that the expected profit from entry are negative, in the absence of information about types.

The key question addressed is the following: provided that such an 'entry deterrence level of investment' exists, can limit pricing deter entry? Moreover, does observability of investment make any difference for the effects of pricing policy? Our major result is that, limit pricing deters entry when investment is *unobservable* and does not when investment is *observable*. In particular, the outcome of the signaling game with observable investment has been shown to be an intuitive separating equilibrium, where the limit price signals to the potential entrant that the incumbent has low costs. The limit price, thus, does not limit entry, because entry does not take place exactly when entry is unprofitable. When investment is unobservable, the most plausible outcome of the signaling game is an intuitive (and Pareto undominated) pooling equilibrium. Any type of incumbent chooses a limit price which does not reveal his cost to the entrant, but credibly conveys the message that investment is above the zero expected profits threshold. As a result, the potential entrant never enters into the market and limit price limits entry when investment is unobservable.

The basic intuition for these results can be summarised as follows. When investment is observable pooling is quite expensive for the low cost incumbent, who is required to make purely dissipative investments. Therefore, separation is preferred to pooling, from the point of view of the low cost incumbent. When investment is unobservable, in contrast, separation requires a greater price distortion, since the high cost incumbent can mimic the low cost one more easily, thus separation is more expensive. Moreover, pooling is cheaper for the low cost incumbent, since no dissipative investment is required. Therefore, with unobservable investment pooling is preferred to separation, not only by the high cost, but also by the low cost incumbent and the pooling equilibrium turns out to be the more plausible outcome.

It must be noticed that in our signaling game with unobservable investment, there are two equilibria surviving the Intuitive Criterion, respectively a pooling and a separating equilibrium. The intuitive pooling equilibrium was selected because it Pareto dominates, from the point of view of the incumbent, the intuitive separating one and also because it satisfies another refinement criterion proposed in the literature. Although we deem the pooling equilibrium the most plausible outcome, we recognize that the thorny issue of selecting among equilibria of a signaling model, with endogenous and unobservable actions affecting the costs of signaling, is a more fundamental problem. A detailed analysis involving the application of more stringent selection criteria is a natural development of the current investigation and is left for future work.

We conclude with a few considerations about implications of our analysis for antitrust policies. This, as it is well known, is a highly debated issue. However, some implications of the above results can be useful.[22] The recent literature on limit pricing was born with the explicit message that limit pricing does not limit entry (Milgrom and Roberts, 1980). Although this view was subsequently softened, the interpretation of signaling models as providing a foundation of the neutrality for entry deterrence of the distortions associated with private information and the associated view that limit pricing should not be part of the antitrust analysis, can be considered common wisdom in many textbooks and overviews and somewhat influential. Our contribution, here, was to show that when the potential entrant is uncertain about the cost of the incumbent firm, a limit price may be a signal that the incumbent has low cost, as

[22]Strictly speaking, the implications are inherently related to which equilibrium is predicted as the outcome of the game. Therefore the importance of the selection criterion mentioned above cannot be understated. Moreover, we did not include any welfare analysis including consumer surplus.

shown by Milgrom and Roberts, but it may as well be a signal that the incumbent is raising a strategic entry barrier through secret actions or investments. In this respect, the analysis of our model preliminary suggests us to lean less towards a "bright line", rule-based, approach to antitrust jurisdiction and to lean more towards a standard case-based approach supported by investigation, dealing with secret investment. This point also is a more proper matter for future work.

Finally, it is worth noticing that the model considered here is an example of a signaling game where both, a hidden characteristic and a hidden action, are present and where the hidden action (effort) affects the cost of the signal, as well as the payoff of the receiver. To our knowledge this class of models has not received great attention and is not been the object of a systematic investigation. Our work provides a first analysis and suggests that in this class of signaling games a pooling equilibrium with a signal conveying information on effort, but not on the hidden characteristic, seems to be a more plausible outcome than that of a separating equilibrium.

Appendix 1

Sketch of the proof of Fact 1.

Let the system of beliefs $\hat{\beta}(e,q)$ satisfy the intuitive criterion. We show that there is no deviation $(\tilde{e},\tilde{q})$ from the equilibrium strategy satisfying (5) and (6). Indeed, if such a deviation exists then $\hat{\beta}(\tilde{e},\tilde{q}) = 1$ and, according to Definition 1.2, firm 2 does not enter at $(\tilde{e},\tilde{q})$. Thus, the system of beliefs cannot support the equilibrium because the equilibrium choice (e^L,q^L) is not optimal, given (6). This contradiction completes the first part of the proof.

To show the converse, let $\hat{\beta}(e,q)$ support a PBE and suppose there is no deviation such that (5) and (6) hold. We have to show that there exists an intuitive system of beliefs satisfying the PBE. This system of beliefs is built by modifying $\hat{\beta}(e,q)$ whenever is needed. Since the deviations equilibrium dominated by H and strictly preferred by L are ruled out by the hypothesis, let us take a deviation from equilibrium, $(\tilde{e},\tilde{q}) \neq (e^t,q^t)$,which is equilibrium dominated for L and strictly preferred by H, i.e. such that

$$V_H(\tilde{e},\tilde{q},0) > V_H(e^H,q^H,y(e^H,q^H)).$$

Since (e^H,q^H) is an equilibrium choice it must be optimal, it thus follows that $V_H(e^H,q^H,y(e^H,q^H)) \geq V_H(\tilde{e},\tilde{q},y(\tilde{e},\tilde{q}))$. Therefore, by the above inequalities it must be true that $y(\tilde{e},\tilde{q}) \neq 0$, hence $y(\tilde{e},\tilde{q}) = 1$. Since firm 2 enters, according to Definition 1.2, we must have $\hat{\beta}(\tilde{e},\tilde{q}) > 0$, and the initial system of beliefs can be possibly modified at $(\tilde{e},\tilde{q})$ to fulfil the intuitive criterion, i.e. by choosing the modified belief $\hat{\beta}'(\tilde{e},\tilde{q}) = 1$. This completes the proof.

Q.E.D.

A preliminary result is needed to prove Lemma 1.

Lemma A1.1 *An incumbent strategy supports a separating equilibrium if and only if* $(e^H,q^H) = (e_A, m_H(e_A))$ *and* (e^L,q^L) *satisfies (9) and (11).*

Proof of Lemma A1.1

Let the incumbent strategy be $(e^H,q^H) = (e_A, m_H(e_A))$ and (e^L,q^L) satisfying (9) and (11). We have to show that it supports a separating equilibrium. Take the beliefs which assign to the H type any choice different from (e^L, q^L), i.e. $\hat{\beta}(e,q) = 1$ for any $(e,q) \neq (e^L, q^L)$ and $\hat{\beta}(e,q) = 0$ for $(e,q) = (e^L, q^L)$. These beliefs obey the Bayes rule. Given these beliefs, firm 2 expected profits are strictly positive except when $(e,q) = (e^L, q^L)$, thus the entrant strategy, $y(e,q) = 0$ if $(e,q) = (e^L, q^L)$ and 1 otherwise, satisfies Definition 1.2. Let us show that given these beliefs the incumbent strategy is optimal. As for type H we have $V_H(e_A, m_H(e_A), 1) \geq V_H(e, q, y(e,q))$ for all (e,q). Indeed, for $(e,q) \neq (e^L, q^L)$, we have $y(e,q) = 1$ and the inequality holds by definition of e_A; moreover, for $(e,q) = (e^L, q^L)$ the above inequality is satisfied because of (9). As for type L, we have $V_L(e^L, q^L, 0) \geq V_L(e, q, y(e,q))$ for all (e,q). Indeed, for $(e,q) \neq (e^L, q^L)$ firm 2 enters, then $y(e,q) = 1$ and the inequality holds because of (11) and definition of V_L^A; if $(e,q) = (e^L, q^L)$ firm 2 does not enter and the above inequality holds as an equality. This completes the first part of the proof.

To show the converse let the incumbent strategy $(e^H, q^H) = (e_A, m_H(e_A))$ and (e^L, q^L) support a separating equilibrium. We have to show that (e^L, q^L) satisfies (9) and (11). Let us suppose, to the contrary, that (e^L, q^L) violates (11); then (e^L, q^L) cannot be an optimal choice for L because $(0, m_L)$ gives higher profits even when Firm 2 enters. Similarly, if (e^L, q^L) violates (9), then $(e_A, m_H(e_A))$ cannot be an optimal choice for type H, since (e^L, q^L) induces no entry and give higher profits. These contradictions complete the proof.

Q.E.D.

Proof of Lemma 1.

Let us suppose that (e^*, q^*) is a solution to the maximization problem. We have to show that the incumbent strategy, $(e^H, q^H) = (e_A, m_H(e_A))$ and $(e^L, q^L) = (e^*, q^*)$, supports an intuitive separating equilibrium. First, notice that (e^*, q^*) satisfies (9) and (11) since these inequalities are equivalent to (13) and (14) respectively. Therefore, by Lemma A1.1, $(e^H, q^H) = (e_A, m_H(e_A))$ and $(e^L, q^L) = (e^*, q^*)$, supports a separating equilibrium. To show that the equilibrium is intuitive, let us suppose that there exists a deviation $(\tilde{e}, \tilde{q}) \neq (e^*, q^*)$, which is equilibrium dominated for H, i.e. $V_H(\tilde{e}, \tilde{q}, 0) < V_H(e_A, m_H(e_A), 1)$, but not for L, i.e. $V_L(\tilde{e}, \tilde{q}, 0) > V_L(e^*, q^*, 0)$. Therefore, $(\tilde{e}, \tilde{q})$ satisfies (13), but from the last inequality $\Pi_L(\tilde{q}) - \tilde{e} > \Pi_L(q^*) - e^*$. Therefore, (e^*, q^*) is not a solution to the maximization problem contrary to the assumption. This contradiction completes the first part of the proof.

To show the converse, let us suppose that the incumbent strategy, $(e^H, q^H) = (e_A, m_H(e_A))$ and $(e^L, q^L) = (e^*, q^*)$, supports an intuitive separating equilibrium. We have to show that (e^*, q^*) is a solution to the maximization problem. Let us proceed by contradiction and suppose that (e^*, q^*) is not a solution, i.e. there exists a pair (e', q') satisfying (13) and (14) such that $\Pi_L(q') - e' > \Pi_L(q^*) - e^*$. From the last inequality it immediately follows that $V_L(e', q', 0) > V_L(e^*, q^*, 0)$. If the constraint (13) is not binding, so that $V_H(e', q', 0) < V_H^A$, the deviation (e', q') violates Fact 1 and $(e^L, q^L) = (e^*, q^*)$ cannot support an intuitive separating equilibrium, contrary to the assumption. Therefore, (13) must be binding so that $V_H(e', q', 0) = V_H^A$. Then, by continuity of the profit functions Π_H and Π_L, one can find in a small neighbourhood of q' a quantity q'' such that $V_L(e', q'', 0) > V_L(e^*, q^*, 0)$ and $V_H(e', q'', 0) < V_H^A$, so that the deviation (e', q'') violates Fact 1 and the equilibrium cannot be intuitive contrary to the assumption. Therefore, we have shown that (e^*, q^*) is a solution to the maximization problem and this completes the proof of the lemma.

Q.E.D.

In order to prove Proposition 1, a preliminary results is needed.

Lemma A1.2 *There exists a unique solution to equation (15), i.e. there exists q^* such that*

$$\Pi_H(0, q^*) + M_H = V_H^A.$$

Moreover, $q^* > m_L$.

Proof of Lemma A1.2.
First of all we know that $V_H(0, q, 0) = \Pi_H(0, q) + M_H$ is continuous in q. By assumption (12), we have $V_H(0, m_L, 0) > V_H^A$. Moreover, let $q_c > m_L$ be the competitive market quantity, i.e. a finite quantity such that $\Pi_L(q_c) = 0$. Then $\Pi_H(0, q_c) < 0$ and $V_H(0, q_c, 0) < M_H < V_H^A$, since $V_H^A \geq M_H + D_H$. Thus by continuity of V_H, equation (15) has at least a solution in the interval $]m_L, q_c[$. Finally, uniqueness of the solution is established by showing that $V_H(0, q, 0)$, or equivalently $\Pi_H(0, q)$, is strictly decreasing for q in the interval $]m_L, q_c[$. This is easily seen by noting that the

profit function is strictly quasi concave and that its maximum is $m_H < m_L$. This completes the proof of Lemma A1.2.

Q.E.D.

Proof of Proposition 1.

By Lemma A1.2, q^* exists and $q^* > m_L$. By Lemma 1, we have to show that $(0, q^*)$ is a solution of the maximization problem, i.e. $\Pi_L(q^*) > \Pi_L(q) - e$ for all $(e, q) \neq (0, q^*)$ satisfying (13) and (14). Let us first consider the maximization problem with only the constraint (13) and separately consider the two cases (e, q) with $q > q^*$ and (e, q) with $q < q^*$. If $q > q^*$ then $q > m_L$ and the profit function $\Pi_L(q)$ is strictly decreasing. Therefore, we have $\Pi_L(q^*) > \Pi_L(q)$ so that $\Pi_L(q^*) > \Pi_L(q) - e$ for all (e, q) with $q > q^*$.

Let us turn to the second case, i.e. (e, q) satisfying (13) with $q < q^*$. By definition of q^*, i.e. by equation (15), the constraint (13) can be rewritten as follows

$$\Pi_H(e, q) - e + M_H(e) \leq \Pi_H(0, q^*) + M_H$$

or

$$\Pi_H(0, q^*) + (M_H - M_H(e)) \geq \Pi_H(e, q) - e.$$

Since monopoly profits are decreasing in costs, $M_H - M_H(e) \leq 0$ for $e \geq 0$, therefore if (e, q) satisfies (13) it also satisfies

$$\Pi_H(0, q^*) \geq \Pi_H(e, q) - e$$

or

$$[p(q^*) - \theta_H]q^* \geq [p(q) - \theta_H(e)]q - e. \tag{24}$$

Moreover, since $q < q^*$ and $\theta_H \geq \theta_H(e)$, it must be true that

$$(\theta_H - \theta_L)q^* \geq (\theta_H(e) - \theta_L)q \tag{25}$$

Adding term by term the inequalities (24) and (25) yields

$$(p(q^*) - \theta_L)q^* > (p(q) - \theta_L)q - e$$

or $\Pi_L(q^*) > \Pi_L(q) - e$ for all (e, q) satisfying (13) with $q < q^*$.

In order to complete the proof we have to show that $(0, q^*)$ also satisfies (14). Notice that, by definition of q^*, we have

$$\Pi_H(0, q^*) + M_H = M_H(e_A) - e_A + D_H(e_A) \tag{26}$$

By definition of V_H^A it must hold

$$M_H(e_A) - e_A + D_H(e_A) \geq M_H + D_H \tag{27}$$

Thus, (26) and (27) imply $\Pi_H(0, q^*) \geq D_H$ and after simple manipulations

$$D_L - D_H \geq D_L - \Pi_H(0, q^*) \tag{28}$$

Next, let us consider

$$\begin{aligned} \Pi_L(q^*) - \Pi_H(0, q^*) &= (\theta_H - \theta_L)q^* \\ &> (\theta_H - \theta_L)m_L \\ &= M_L - \Pi_H(0, m_L) \\ &> M_L - M_H \end{aligned} \tag{29}$$

where the first inequality follows from $q^* > m_L$ and the last inequality from $M_H > \Pi_H(0, m_L)$. By assumption (4) for $e = 0$, we have $M_L - M_H \geq D_L - D_H$ so that from (28) and (29) it follows

$$\Pi_L(q^*) - \Pi_H(0, q^*) > D_L - \Pi_H(0, q^*)$$

and finally $\Pi_L(q^*) > D_L$. This completes the proof of Proposition 1.

Q.E.D.

Proof of Lemma 2

Let (e^P, q^P) satisfy (16) and (17) with $e^P \geq e_0$. We have to show that it supports a pooling equilibrium. Let us take the beliefs $\hat{\beta}(e, q) = \beta$ if $(e, q) = (e^P, q^P)$ and 1 otherwise. The beliefs are consistent with Bayes rule, Definition 1.3. Given the beliefs, take the entrant strategy $y(e, q) = 0$ if $(e, q) = (e^P, q^P)$ and 1 otherwise. The incumbent strategy is optimal for type H. In fact, if $(e, q) \neq (e^P, q^P)$ then

$y(e,q) = 1$ and, by definition of V_H^A, we have $V_H^A \geq V_H(e,q,y(e,q))$ and finally, by (16), $V_H(e^P,q^P,0) \geq V_H(e,q,y(e,q))$. With a similar argument it is easily shown that (e^P,q^P) is optimal for type L, so that the incumbent strategy satisfies Definition 1.1. Let us show that also the entrant strategy is optimal. If $y(e,q) = 1$, then $(e,q) \neq (e^P,q^P)$ and $\hat{\beta}(e,q) = 1$, thus the entrant expected profits are strictly positive. Viceversa, if at (e,q) the expected profits are strictly positive then $(e,q) \neq (e^P,q^P)$, because $\hat{\beta}(e^P,q^P) = \beta$ and $e^P \geq e_0$ imply, by assumption A.5 non positive expected entrant profits. Thus, $(e,q) \neq (e^P,q^P)$ and by the definition of y we have $y(e,q) = 1$ and Definition 1.2 is satisfied.

Let us show the converse an suppose that $(e^H,q^H) = (e^L,q^L) = (e^P,q^P)$ supports a pooling equilibrium. Then, by Definition 1.3, $\hat{\beta}(e^P,q^P) = \beta$ and $y(e^P,q^P) = 0$, because if $y(e^P,q^P) = 1$ the pooling strategy would not be an optimal strategy for the incumbent. Therefore, the entrant expected profits must be non positive, which means, by assumption A.5, that $e^P \geq e_0$. Consider next the choice of L. Since by Definition 1.1 $V_L(e^P,q^P,0) \geq V_L(0,m_L,y(0,m_L))$ and since $V_L(0,m_L,0)$ is the highest total profit, it must be $y(0,m_L) = 1$ thus $V_L(e^P,q^P,0) \geq V_L(0,m_L,1) = V_L^A$ and (17) holds. Consider the choice of type H. Since by Definition 1.1 $V_H(e^P,q^P,0) \geq V_H(e_A,m_H(e_A),y(e_A,m_H(e_A)))$ and since $V_H(e_a,m_H(e_A),y(e_A,m_H(e_A))) \geq V_H^A$ then $V_H(e^P,q^P,0) \geq V_H^A$ and (16) is satisfied. This completes the proof of Lemma 2.

Q.E.D.

Proof of Proposition 2.
Let (e^P,q^P) be the pooling equilibrium strategy of the incumbent. We show that there exists a deviation $(0,\tilde{q})$ which is equilibrium dominated for the H type and strictly preferred to the equilibrium choice by the L type.

Let $\tilde{q} > m_L$ be defined by the equality

$$\Pi_L(q^P) - \Pi_L(\tilde{q}) = e^P - \epsilon \tag{30}$$

where $\epsilon > 0$ is arbitrarily close to zero so that $e^P - \epsilon > 0$. The quantity $\tilde{q}$ is well defined and $\tilde{q} > \max\{m_L, q^P\}$.[23] Let us consider the deviation $(0,\tilde{q})$ and show that

[23]Let us consider the function of q, $\Pi_L(q) - [\Pi_L(q^P) - e^P + \epsilon]$ and notice that by (17) the term in

it is strictly preferred to the equilibrium choice by the L type. Indeed,

$$\begin{aligned} V_L(0,\tilde{q},0) - V_L(e^P,q^P,0) &= \Pi_L(\tilde{q}) + M_L - \Pi_L(q^P) + e^P - M_L \\ &= \Pi_L(\tilde{q}) - \Pi_L(q^P) + e^P \\ &= \epsilon - e^P + e^P = \epsilon > 0 \end{aligned}$$

where we used (30).

Before showing that the same deviation is equilibrium dominated for the H type let us derive the following result.

$$\begin{aligned} \Pi_H(e^P,q^P) - \Pi_H(0,\tilde{q}) &- \Pi_L(q^P) + \Pi_L(\tilde{q}) = \\ &= -\theta_H(e^P)q^P + \theta_H\tilde{q} + \theta_L q^P - \theta_L\tilde{q} \\ &= (\theta_H - \theta_L)(\tilde{q} - q^P) + (\theta_H - \theta_H(e^P))q^P > 0 \end{aligned} \quad (31)$$

where the second equality follows by adding and subtracting $\theta_H q^P$ and the last inequality from $\tilde{q} > q^P$. By (30) and (31) we have

$$\Pi_H(0,\tilde{q}) - \Pi_H(e^P,q^P) + e^P < \epsilon$$

Subtracting to both sides $M_H(e^P) - M_H > 0$ yields

$$\Pi_H(0,\tilde{q}) + M_H - \Pi_H(e^P,q^P) + e^P - M_H(e^P) < \epsilon - [M_H(e^P) - M_H] \quad (32)$$

where the LHS is the variation of type H total profits after the deviation $(0,\tilde{q})$, i.e.

$$V_H(0,\tilde{q},0) - V_H(e^P,q^P,0) = \Pi_H(0,\tilde{q}) + M_H - \Pi_H(e^P,q^P) + e^P - M_H(e^P). \quad (33)$$

(32) and (33) give

$$\begin{aligned} V_H(0,\tilde{q},0) - V_H(e^P,q^P,0) &< \epsilon - [M_H(e^P) - M_H] \\ &< 0 \end{aligned}$$

square brackets is strictly positive. Clearly, for $q = m_L$ the function is strictly positive and at the perfectly competitive quantity, q_c, the above function is strictly negative since $\Pi_L(q_c) = 0$. Thus, by continuity there exists $\tilde{q}$ satisfying (30) in the open interval $]m_L, q_c[$. Moreover, since for $q > m_L$ the profit function is strictly decreasing, $\tilde{q}$ is unique. Finally, if $q^P > m_L$, by inspection of (30) it is easily seen that $\tilde{q} > q^P$.

where the last inequality follows from the fact the the term in square brackets is strictly positive and ϵ can be taken arbitrarily close to zero. Therefore, we have shown that the deviation $(0, \tilde{q})$ is equilibrium dominated for the H type. Since for any pooling equilibrium one can find a deviation which is equilibrium dominated for the H type and strictly preferred by the L type, by Fact 1 there exists no pooling equilibrium satisfying the intuitive criterion and this completes the proof of Proposition 2.

Q.E.D.

Appendix 2

This appendix contains the proofs of the results of Section 4. Let us first prove the properties of the functions $\phi(q)$ and $U(q)$ which are extensively used in the analysis of this section.

Lemma A2.0. *Let $\bar{e} = argmax_e\, V_H(e, m_H(e), 0)$ and $\bar{m} = m_H(\bar{e})$.*

(i) The investment function $\phi(q)$ is strictly increasing and $\phi(\bar{m}) = \bar{e}$.

(ii) The value function $U(q)$ has a global maximum at $q = \bar{m}$, is strictly increasing for $q \leq \bar{m}$ and strictly decreasing for $q \geq \bar{m}$.

Proof of Lemma A2.0.(i)

Let $q'' > q'$, $e'' = \phi(q'')$ and $e' = \phi(q')$. We have to show that $\phi(q'') > \phi(q')$. By definition of e' and e'' we have

$$\begin{aligned} \Pi_H(e', q') - e' + M_H(e') &\geq \Pi_H(e'', q') - e'' + M_H(e'') \\ \Pi_H(e'', q'') - e'' + M_H(e'') &\geq \Pi_H(e', q'') - e' + M_H(e') \end{aligned} \tag{34}$$

Adding the two inequalities yields

$$\Pi_H(e'', q'') + \Pi_H(e', q') \geq \Pi_H(e', q'') + \Pi_H(e'', q')$$

By rearranging we obtain

$$\Pi_H(e'', q'') - \Pi_H(e', q'') \geq \Pi_H(e'', q') - \Pi_H(e', q')$$

and then $[\theta_H(e') - \theta_H(e'')](q'' - q') \geq 0$. Since $q'' > q'$, we have $\theta_H(e') \geq \theta_H(e'')$ and since θ_H is strictly decreasing, we obtain $e'' \geq e'$. Since, by assumption A.2 $e' \neq e''$, we have $e'' > e'$ or, by definition of e' and e'', $\phi(q'') > \phi(q')$, so that the investment function $\phi(q)$ is strictly increasing.

Next, let us show that $\phi(\bar{m}) = \bar{e}$ and suppose that there exists $e' \neq \bar{e}$ such that $e' = \phi(\bar{m})$. By definition of e', it holds $V_H(e', \bar{m}, 0) > V_H(\bar{e}, \bar{m}, 0)$ then

$$\Pi_H(e', \bar{m}) - e' + M_H(e') > 2M_H(\bar{e}) - \bar{e} \tag{35}$$

Since $\Pi_H(e', m_H(e')) \geq \Pi_H(e', \bar{m})$, by (35) we have $2M_H(e') - e' > 2M_H(\bar{e}) - \bar{e}$ which contradicts the assumption that $\bar{e}$ maximizes $V_H(e, m_H(e), 0)$. Thus, $V_H(\bar{e}, \bar{m}, 0) \geq V_H(e, \bar{m}, 0)$ for all e or $\bar{e} = \phi(\bar{m})$.

Proof of Lemma A2.0.(ii)
Let us notice first that

$$V_H(e, m_H(e), 0) \geq V_H(e, q, 0) \tag{36}$$

for all e and q. Indeed, (36) is equivalent to the inequality $\Pi_H(e, m_H(e)) \geq \Pi_H(e, q)$, which is clearly satisfied. Moreover, by Lemma A2.0.(i) and the definition of $\bar{e}$ we have

$$U(\bar{m}) = V_H(\phi(\bar{m}), \bar{m}, 0) = V_H(\bar{e}, \bar{m}, 0) \geq V_H(e, m_H(e), 0) \tag{37}$$

for all e. Therefore, (36) and (37) give $U(\bar{m}) \geq V_H(e, q, 0)$ for all e and q. Since this also holds $e = \phi(q)$, we have $U(\bar{m}) \geq U(q)$ for all q, i.e. $\bar{m}$ is a global maximum for $U(q)$.

In order to prove the rest of Lemma A2.0.(ii) we need a preliminary result.

Lemma A2.1 *Let $q' \in (m_H, m_L)$, $\bar{m} = m_H(\bar{e})$ and $e' = \phi(q')$. The following results hold: (i) If $q' < \bar{m}$ then $q' \leq m_H(e') < \bar{m}$. (ii) If $q' > \bar{m}$ then $\bar{m} < m_H(e') \leq q'$.*

Proof of Lemma A2.1. If $q' = m_H(e')$, (i) and (ii) hold. Thus, let $q' \neq m_H(e')$. From $q' > m_H$ and strict monotonicity of $\phi(q)$, we obtain $e' > 0$ and then $m_H(e') > m_H$.

Let $e'' = \phi(m_H(e'))$, then, from $q' \neq m_H(e')$ and strict monotonicity of $\phi(q)$, we have $e'' \neq e'$ and

$$V_H(e'', m_H(e'), 0) > V_H(e', m_H(e'), 0) \tag{38}$$

Moreover, $m_H(e'') \neq m_H(e')$ and $\Pi_H(e'', m_H(e'')) > \Pi_H(e'', m_H(e'))$, thus we have

$$V_H(e'', m_H(e''), 0) > V_H(e'', m_H(e'), 0) \tag{39}$$

From (38) and (39) we obtain

$$V_H(e'', m_H(e''), 0) > V_H(e', m_H(e'), 0) \tag{40}$$

(i) If $q' < \bar{m}$, then, by Lemma A2.0.(i), $e' < \bar{e}$ and $m_H(e') < \bar{m}$. Next, let us suppose that $m_H(e') < q'$, then, by Lemma A2.0.(i), $e'' < e'$ and since $V_H(e, m_H(e), 0)$ is increasing for $e < \bar{e}$ we obtain $V_H(e'', m_H(e''), 0) \leq V_H(e', m_H(e'), 0)$, which contradicts (40). Therefore $m_H(e') \geq q'$ and since $q' \neq m_H(e')$ we have $m_H(e') > q'$.

(ii) If $q' > \bar{m}$, then $e' > \bar{e}$ and $m_H(e') > \bar{m}$. Next, let us suppose that $m_H(e') > q'$, then $e'' > e'$ and since $V_H(e, m_H(e), 0)$ is decreasing for $e > \bar{e}$ we obtain $V_H(e'', m_H(e''), 0) \leq V_H(e', m_H(e'), 0)$, which contradicts (40). Therefore $m_H(e') \leq q'$ and since $q' \neq m'$ we have $m_H(e') < q'$. This completes the proof of Lemma A2.1.

Let us go back to the proof of Lemma A2.0.(ii) and show that $U(q)$ is strictly increasing for $q \in [m_H, \bar{m}]$. Let $q'' > q'$, $e' = \phi(q')$ and $e'' = \phi(q'')$. By Lemma A2.1(i) we have $q' \leq m_H(e')$. There are two cases, (a) $q'' \leq m_H(e')$ and (b) $q' \leq m_H(e') < q''$.

(a) If $q'' \leq m_H(e')$, by the properties of the profit function we have $\Pi_H(e', q') < \Pi_H(e', q'')$. Adding to both sides $M_H(e') - e'$ yields

$$U(q') < V_H(e', q'', 0) \tag{41}$$

Moreover, since $e'' = \phi(q'')$ we have

$$V_H(e', q'', 0) \leq V_H(e'', q'', 0) = V_H(\phi(q''), q'', 0) = U(q'') \tag{42}$$

Therefore, (41) and (42) give the result $U(q'') > U(q')$.

(b) If $q' \leq m_H(e') < q''$ we apply the following argument. By Lemma A2.1.(i), $m_H(e'') \geq q''$, thus since $m_H(e') < q'' \leq m_H(e'')$ and $m_H(e)$ is continuous, there exists $e' < \hat{e} \leq e''$ such that $m_H(\hat{e}) = q''$. Therefore, since $V_H(e, m_H(e), 0)$ is strictly increasing for $e < \bar{e}$ we have

$$V_H(e', m_H(e'), 0) < V_H(\hat{e}, m_H(\hat{e}), 0) \tag{43}$$

Moreover, we know that

$$V_H(e', q', 0) \leq V_H(e', m_H(e'), 0) \tag{44}$$

since $\Pi_H(e', q') \leq \Pi_H(e', m_H(e'))$. Also

$$V_H(\hat{e}, q'', 0) \leq V_H(e'', q'', 0) \tag{45}$$

since $e'' = \phi(q'')$. Thus (43), (44) and (45) give

$$U(q') = V_H(e', q', 0) < V_H(e'', q'', 0) = U(q'').$$

The proof that $U(q)$ is strictly decreasing for $q \geq \bar{m}$ is similar. If $q' \geq m_L$, the inequality $m_H(e'') < q'$ holds, since by assumption A.1 $m_H(e) < m_L$ for all e. Then, $\Pi_H(e'', q') > \Pi_H(e'', q'')$ since $q'' > q'$ and Π_H is strictly decreasing for quantities greater than the monopoly level $m_H(e'')$. Adding $M_H(e'') - e''$ to both sides yields $\Pi_H(e'', q') - e'' + M_H(e'') > U(q'')$ and, finally, by definition of $U(q')$ we obtain $U(q') > U(q'')$.

Let us show that $U(q)$ is strictly decreasing for $q \in [\bar{m}, m_L]$. Let $\bar{m} \leq q' < q'' \leq m_L$. By Lemma A2.1.(ii), we have $m_H(e'') \leq q''$. There are two cases. (a) If $m_H(e'') \leq q'$ we apply the above argument. (b) If $q' < m_H(e'')$, we argue as follows. By Lemma A2.1.(ii), $m_H(e') \leq q'$, therefore we have $m_H(e') \leq q' < m_H(e'')$. By continuity of $m_H(e)$, there exists $e' \leq \hat{e} < e''$ such that $m_H(\hat{e}) = q'$. Since $V_H(e, m_H(e), 0)$ is strictly decreasing for $e > \bar{e}$ we have

$$V_H(e'', m_H(e''), 0) < V_H(\hat{e}, m_H(\hat{e}), 0) \tag{46}$$

Moreover, we know that

$$V_H(e'', q'', 0) \leq V_H(e'', m_H(e''), 0) \tag{47}$$

since $\Pi_H(e'', q'') \leq \Pi_H(e'', m_H(e''))$. Also

$$V_H(\hat{e}, q', 0) \leq V_H(e', q', 0) \tag{48}$$

since e' is the best investment given q'. Thus (46), (47) and (48) give

$$U(q'') = V_H(e'', q'', 0) < V_H(e', q', 0) = U(q')$$

Finally, if $q' < q''$ with $q' < m_L < q''$ we apply the above results to show that $U(q') > U(m_L) > U(q'')$. This completes the proof of Lemma A2.0.

Q.E.D.

In order to prove Lemma 3 we need the following preliminary result.

Lemma A2.2 *An incumbent strategy* (e^H, q^H), (e^L, q^L) *supports a separating equilibrium if and only if* $e^H = e_A$, $q^H = m_H(e_A)$, $e^L = 0$ *and* q^L *satisfies (20) and (21).*

The proof of Lemma A2.2 is similar to the proof of Lemma A1.1 and will be omitted.

Proof of Lemma 3.
Let us suppose that q' is a solution to the maximization problem. By Lemma A2.2, the choice $(e^L, q^L) = (0, q')$ supports a separating equilibrium, since $(0, q')$ satisfies (20) and (21). To show that the separating equilibrium also satisfies the intuitive criterion, let us suppose that there exists a deviation $\tilde{q}$ such that $U(\tilde{q}) < V_H^A$ and $V_L(0, \tilde{q}, 0) > V_L(0, q', 0)$. But, then $\Pi_L(\tilde{q}) > \Pi_L(q')$ and q' can not be a solution to the maximization problem. This contradiction completes the first part of the proof.

Let us show the converse and suppose that (e^L, q^L) supports an intuitive separating equilibrium. We have to show that $e^L = 0$ and that q^L is a solution to the maximization problem. First, notice that $e > 0$ is a dominated choice for the type L so that the optimal choice must be $e^L = 0$. Next, let us proceed by contradiction and suppose that q^L is not a solution, i.e. there exists $q' \neq q^L$ such that $\Pi_L(q') > \Pi_L(q^L)$ and $U(q') \leq V_H^A$. Thus, it follows that $V_L(0, q', 0) > V_L(0, q^L, 0)$ and $q' \geq m_L$.[24] If

[24]Indeed, by the mimicking condition (12) and $U(q) \geq V_H(0, q, 0)$, we have $U(m_L) \geq V_H^A$. Thus, by monotonicity of $U(q)$, the inequality $U(q') \leq V_H^A$ implies $q' \geq m_L$.

$U(q') < V_H^A$ the equilibrium is not intuitive, contrary to the assumption, therefore, it must be $U(q') = V_H^A$. Since q^L supports a separating equilibrium, by Lemma A2.2, $U(q^L) \leq V_H^A$. Thus $q^L > m_L$ and $U(q^L) \leq U(q')$. Since $q' \neq q^L$ and $U(q)$ is strictly decreasing (by Lemma A2.0.(ii)), we must have $U(q^L) < U(q')$ so that $q^L > q'$. By continuity of Π_L and U, there exists $q'' > q'$ sufficiently close to q' such that $U(q'') < U(q') = V_H^A$ and $\Pi_L(q'') > \Pi_L(q^L)$ or $V_L(0, q'', 0) > V_L(0, q^L, 0)$. Therefore, $(0, q^L)$ does not support an intuitive equilibrium, contrary to the assumption. This contradiction completes the proof.

Q.E.D.

The next result is needed to prove Proposition 3.

Lemma A2.3 *A solution to the equation $U(q) = V_H^A$ exists and is unique in the interval $[m_L, q_c]$. The solution, denoted by q^{**}, satisfies the condition $q^{**} > q^*$, where q^* is the intuitive separating equilibrium quantity with observable investment.*

Proof of Lemma A2.3. Since $U(q)$ is continuous and, by Lemma A2.0.(ii), strictly decreasing in the interval $[m_L, q_c]$, it is sufficient to show that $U(m_L) > V_H^A$ and $U(q_c) < V_H^A$. By (12), we have $V_H(0, m_L, 0) > V_H^A$, thus by definition of $U(q)$ we obtain $U(m_L) > V_H^A$. Next, since $\theta_H(e) > \theta_L$ for all $e \geq 0$, we have $\Pi_H(e, q_c) < \Pi_L(q_c) = 0$. Since the incumbent's duopoly profits are positive, i.e. $D_H(e) > 0$, we can write $\Pi_H(e, q_c) < D_H(e)$ for all $e \geq 0$. Adding to both sides $M_H(e) - e$ yields $V_H(e, q_c, 0) < M_H(e) - e + D_H(e)$, from which follows $U(q_c) < V_H^A$.

Finally, we show that $q^{**} > q^*$. Notice that $U(q^*) > V_H(0, q^*, 0) = V_H^A = U(q^{**})$, where the first inequality follows from the definition of V and the hypothesis that investment is desirable. The equalities follow from the definitions of q^* and q^{**}. Finally, by Lemma A2.0.(ii), $U(q^{**}) < U(q^*)$ implies $q^{**} > q^*$. This completes the proof of Lemma A2.3.

Q.E.D.

Proof of Proposition 3.
By Lemma A2.3, q^{**} exists, is unique and $q^{**} > q^*$. By Lemma 3, we have to show

that q^{**} is a solution to the following maximization problem

$$\begin{array}{ll} \max\limits_{q} & \Pi_L(q) \\ \text{subject to} & U(q) \leq V_H^A \\ & \Pi_L(q) \geq D_L \end{array}$$

If $m_L < q < q^{**}$, by Lemma A2.0.(ii), $U(q) > U(q^{**}) = V_H^A$ and q does not satisfy the first constraint. If $q > q^{**}$ then $\Pi_L(q^{**}) \geq \Pi_L(q)$, since Π_L is decreasing for $q \geq m_L$. Thus q^{**} maximizes Π_L subject to the first constraint. To complete the proof we have to show that q^{**} also satisfies the constraint $\Pi_L(q^{**}) \geq D_L$. In fact, notice that, by definition of q^{**}, we have

$$\Pi_H(e^{**}, q^{**}) - e^{**} + M_H(e^{**}) = M_H(e_A) - e_A + D_H(e_A), \tag{49}$$

where $e^{**} = \phi(q^{**})$. By definition of V_H^A it must hold

$$M_H(e_A) - e_A + D_H(e_A) \geq M_H(e^{**}) - e^{**} + D_H(e^{**}). \tag{50}$$

Thus, (49) and (50) imply $\Pi_H(e^{**}, q^{**}) \geq D_H(e^{**})$ and after simple manipulations

$$D_L - D_H(e^{**}) \geq D_L - \Pi_H(e^{**}, q^{**}). \tag{51}$$

Next, notice that

$$\begin{aligned} \Pi_L(q^{**}) - \Pi_H(e^{**}, q^{**}) &= (\theta_H(e^{**}) - \theta_L)q^{**} \\ &> (\theta_H(e^{**}) - \theta_L)m_L \\ &= M_L - \Pi_H(e^{**}, m_L) \\ &> M_L - M_H(e^{**}), \end{aligned} \tag{52}$$

where the first inequality follows from $q^{**} > m_L$ and the last inequality from $M_H(e^{**}) > \Pi_H(e^{**}, m_L)$. From condition (4) we have

$$M_L - M_H(e^{**}) \geq D_L - D_H(e^{**}),$$

therefore (51) and (52) yield

$$\Pi_L(q^{**}) - \Pi_H(e^{**}, q^{**}) > D_L - \Pi_H((e^{**}, q^{**})$$

and, finally, $\Pi_L(q^{**}) > D_L$. This completes the proof of Proposition 3.

Q.E.D.

Proof of Lemma 4.
Let the incumbent strategy satisfy (i), (ii) and (iii). We have to show that it supports a pooling equilibrium. Let us set the beliefs as follows: $\hat{\beta}(q) = \beta$ if $q = q^P$ and 1 otherwise. Moreover, let $\hat{e}(q) = e^H$ if $q = q^P$ and e_A otherwise. Therefore, beliefs and conjectures satisfy Definition 2.3. Given the above beliefs, the entrant strategy, $y(q) = 0$ if $q = q^P$ and 1 otherwise, is optimal according to Definition 2.2, since, by (iii), $e^H \geq e_0$. As to the incumbent strategy, let us show that the L type choice is optimal. In fact, for $q \neq q^P$, $V_L(e, q, y(q)) = V_L(e, q, 1) \leq V_L^A$. Thus, by (ii), $V_L(0, q^P, 0) \geq V_L(e, q, y(q))$ and the L type choice is optimal. Similarly, for $q \neq q^P$, $V_H(e, q, y(q)) = V_H(e, q, 1)$ and, by definition of V_H^A, $V_H^A \geq V_H(e, q, 1)$. By (iii), $e^H = \phi(q^P)$ so that $V_H(e^H, q^P, 0) = U(q^P)$. Therefore, by (i) and the last inequality we have $V_H(e^H, q^P, 0) \geq V_H(e, q, y(q))$ and the H type choice is optimal. This completes the first part of the proof.

To show the converse, let the incumbent strategy support a pooling equilibrium. We have to show that (i), (ii) and (iii) hold. Since e^H and q^P support a pooling equilibrium, by Definition 2.3, we have $\hat{\beta}(q^P) = \beta$ and $\hat{e}(q^P) = e^H$. It must hold $e^H \geq e_0$, otherwise the entrant optimal strategy is $y(q) = 1$ for all q and q^P cannot be optimal for both the types of incumbent. Therefore, $e^H \geq e_0$ and $y(q^P) = 0$. Let us consider the L type choice. By optimality of q^P we have $V_L(0, q^P, 0) \geq V_L(0, m_L, y(m_L)) \geq V_L^A$ and (ii) holds. Similarly, $V_H(e^H, q^P, 0) \geq V_H(e_A, m_H(e_A), y(m_H(e_A))) \geq V_H^A$. Moreover, $e^H = \phi(q^P)$ otherwise (e^H, q^P) can not be an optimal choice for H. Therefore, (iii) holds and since $V_H(e^H, q^P, 0) = U(q^P)$ also (i) is satisfied, and this completes the proof.

Q.E.D.

Proof of Proposition 4.
Using Lemma 5 we show that if $q_0 < q^{**}$ there exists e^H and q^P, with $\hat{q} \leq q^P \leq q^{**}$, satisfying (i), (ii) and (iii), where $\hat{q} = q_0$, if $q_0 > m_L$, and $\hat{q} = m_L$ otherwise. Take $e^H = \phi(q^P)$ and notice that (iii) holds, since by Lemma A2.0.(i) the function ϕ is

increasing and $e^H = \phi(q^P) \geq \phi(q_0) = e_0$ since $q^P \geq \hat{q} \geq q_0$. That $U(q^P) > V_H^A$ is easily seen by noting that $U(m_L) > V_H^A$, $U(q^{**}) = V_H^A$ and the value function U is decreasing, so condition (i) of Lemma 4 also holds. Finally, (ii) trivially holds because $V_L(0, q^{**}, 0) \geq V_L^A$ and $V_L(0, q, 0)$ is decreasing for $q \geq m_L$. Therefore, we have shown that q^P, with $\hat{q} \leq q^P \leq q^{**}$, supports a pooling equilibrium.

To prove Proposition 4.(i) we have to show that $q^P < \hat{q}$ can not support an intuitive pooling equilibrium. If $q_0 > m_L$, then $\hat{q} = q_0$ and according to Lemma 4 there is no Pooling equilibrium supported by $q^P < q_0$. Thus let $q_= < m_L$ and take a pooling equilibrium supported by q^P with $q_0 \leq q^P < m_L$. It is easily seen that the deviation $\tilde{q} = m_L$ is equilibrium dominated for type H and strictly preferred by type L so that q^P can not support an intuitive equilibrium.

To prove Proposition 4.(ii), consider the pooling equilibrium supported by $\hat{q}$ and show that it satisfies the intuitive criterion. If $q_0 \leq m_L$, the quantity $\hat{q} = m_L$ supports an intuitive pooling equilibrium; indeed, there is no deviation from m_L which is strictly preferred by the low cost incumbent since m_L maximizes monopoly profits.

If $q_0 > m_L$ then any deviation $\tilde{q} > q_0$ is equilibrium dominated for both types. A deviation $\tilde{q} < q_0$ has to be analysed more carefully and two cases are to be considered.

(a) If $U(m_H) \geq U(q_0)$, then, by Lemma A2.0.(ii), $U(\tilde{q}) \geq U(q_0)$ and there is no deviation equilibrium dominated for type H so that, according to Fact 2, $\hat{q} = q_0$ satisfies the intuitive criterion.

(b) If $U(m_H) < U(q_0)$ an additional mild assumption is needed which requires that the low cost incumbent profits are greater for quantities close to m_L, rather than for quantities close to m_H. More formally, let us consider the following assumption.

A.6 Let $q_0 > m_L$. If $U(m_H) < U(q_0)$ then $q' < q''$, where $q' \neq q_0$ is given by $U(q_0) = U(q')$ and $q'' \neq q_0$ is given by $V_L(0, q_0, 0) = V_L(0, q'', 0)$.

The quantities q' and q'' are well defined and are shown in Figure 6. If Assumption A.6 holds any deviation $\tilde{q} < q_0$ which is equilibrium dominated for type H cannot be strictly preferred by type L, since both the inequalities $\tilde{q} < q'$ and $\tilde{q} > q''$ cannot

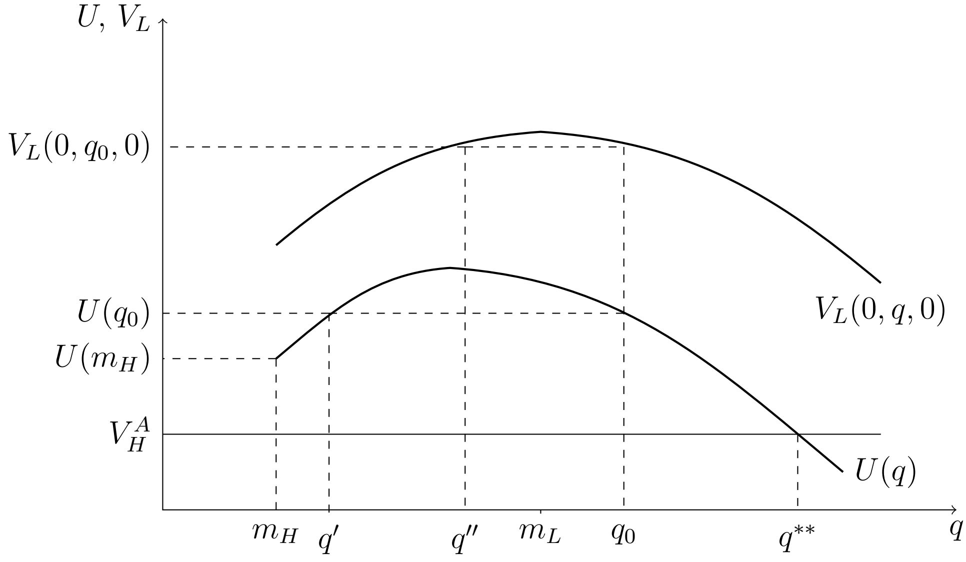

Figure 6: Quantities q' and q'' in Assumption A.6

be satisfied. Therefore, we can conclude that $\hat{q} = q_0$ supports an intuitive pooling equilibrium.

Finally, to see that the quantity $\hat{q}$ supports the unique Pareto undominated intuitive pooling equilibrium, it is sufficient to notice that both functions, $U(q)$ and $V_L(0, q, 0)$ are strictly decreasing for $q \geq m_L$ and that intuitive pooling equilibrium quantities must be greater than $\hat{q}$. Therefore, if $q^P \neq \hat{q}$ supports an intuitive pooling equilibrium, then $U(\hat{q}) > U(q^P)$ and $V_L(0, \hat{q}, 0) > V_L(0, q^P, 0)$ since $q^P > \hat{q}$.

Q.E.D.

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